Drainage Correlation Research Project

INTERIM REPORT #7
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FREQUENCY OF PEAK FLOWS PREDICTED FROM RAINFALL FREQUENCIES

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ENGINEERING

Research Laboratories

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Prepared for

Montana State Highway Commission

and

U. S. Bureau of Public Roads

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ABSTRACT

A theoretical equation is derived which may be used to predict floods on small Montana watersheds. The equation derived uses the mean annual peak discharge rate of the stream (which may be estimated from short streamflow records) together with three theoretically based factors. These three factors were evaluated for Montana east of the continental divide.

The rainfall intensity ratio, defined as the ratio of the 50-year rainfall intensity of given duration to the mean annual rainfall intensity, of the same duration, was evaluated using published rainfall data. An isoplethal map was plotted showing values of the rainfall intensity ratio for Montana east of the continental divide.

The rainfall-discharge recurrence factor, a factor expressing the relationship between the rainfall intensity recurrence relation and the recurrence relation of peak annual discharges produced by rainfall, was evaluated using published rainfall data, as well as water-stage and discharge data for twelve Montana watersheds. Special techniques were required to separate rainfall-induced flow from the remaining portions of streamflow hydrographs so that peak annual discharges from rainfall could be determined. Values of the rainfall discharge-recurrence factor determined for the twelve watersheds were used to plot an isoplethal map applicable to Montana east of the continental divide.

The rain-snow-base flow interaction ratio, a factor reflecting the relative dependence or independence of the frequency curves of rainfall-induced flows and snow-melt-induced flows, was evaluated for the twelve Montana watersheds. An average value is recommended for use in Montana east of the continental divide.

Estimates of the 50-year flood obtained by extreme value (Gumbel) and log-normal recurrence analyses were compared with estimates from the method derived in this study for 46 watersheds. From this comparison, the derived method is considered to be more reliable than the extreme value and log-normal methods for discharge records 50 years or shorter in length.

The derived method is applicable for watersheds less than 500 square miles in area for which large events occur most often during the snow melt season.

CHAPTER I

INTRODUCTION

As a result of the intensive highway construction program which is currently in progress in Montana, larger and more expensive drainage structures are being built on small Montana watersheds than have been in the past. As the size and cost of these structures increases, the need for reliable estimates of the magnitude of floods which may occur during the lifetime of the structures also increases.

Predicting floods on small watersheds in Montana is difficult. Only since the inception of the U.S. Geological Survey Small-area Peak-flow Highway Program in 1955 have peak-flow measurements been made on more than a few such watersheds. These records are generally too short to be used without additional information in predicting floods of more than very short recurrence intervals. No satisfactory flood-prediction methods using data other than streamflow records have been developed for use in Montana.

Precipitation measurements, on the other hand, have been made for many years at stations located throughout Montana. Since peak flow rates are related to rainfall, at least in part, a logical approach to predicting floods is to utilize the long rainfall records which are available. This approach is taken as one phase of the Drainage Correlation Research Project.

The Drainage Correlation Research Project, sponsored by the Montana State Highway Commission and the Bureau of Public Roads, was instituted at Montana State University in 1963 to develop and evaluate procedures for predicting floods from Montana watersheds 1 to 100 square miles in area. The investigation is divided into two concurrent phases:

1) Runoff data from a number of watersheds and precipitation data from weather stations throughout Montana are studied to develop a correlation between peak flows and precipitation data.

2) Comprehensive hydrologic studies are being made of four watersheds included in the U.S. Geological Survey Peak-flow Program. Data collected is used to develop and evaluate flood prediction procedures.

The research reported herein was conducted under the first phase of the project investigation.

In this study a theoretical equation is derived which may be used to predict floods on small Montana watersheds. The equation derived uses the mean annual peak discharge rate of the stream (which may be estimated from short streamflow records) together with three theoretically based factors. These three factors were evaluated for Montana east of the continental divide.

The rainfall intensity ratio, defined as the ratio of the 50-year rainfall intensity of given duration to the mean annual rainfall intensity of the same duration, was evaluated using published rainfall data. An

isoplethal map was plotted showing values of the rainfall intensity ratio for Montana east of the continental divide.

The rainfall-discharge recurrence factor, a factor expressing the relationship between the rainfall intensity recurrence relation (for specified durations) and the recurrence relation of peak annual discharges produced by rainfall, was evaluated using published rainfall data and water-stage and discharge data for twelve Montana watersheds. Special techniques were required to separate rainfall-induced flow from the remaining portions of streamflow hydrographs so that peak annual discharges from rainfall could be determined. Values of the rainfall discharge-recurrence factor determined for the twelve watersheds were used to plot an isoplethal map applicable to Montana east of the continental divide.

The rain-snow-base flow interaction ratio, a factor reflecting the relative dependence or independence of the frequency curves of rainfall-induced flows and snow-melt-induced flows, was evaluated for the twelve Montana watersheds. An average value is recommended for use in Montana east of the continental divide.

Data for developing the method was obtained from twelve water-stage-gaged watersheds in Montana east of the continental divide. The method was tested on a number of water-stage-gaged and crest-stage-gaged watersheds located east of the continental divide.

CHAPTER II

REVIEW OF LITERATURE

Since 1940 wide scale development of hydrologic techniques applicable to small watersheds has taken place. Some of the most useful developments have been in the application of frequency analyses. Below, traditional and modern techniques of flood prediction which incorporate rainfall data are outlined; the current methods of frequency analysis are discussed; and the methods available for predicting floods on small Montana watersheds are presented.

RAINFALL-RELATED FLOOD FORMULAS

The rational runoff formula, proposed and used by Mulvany as early as 1851, (Dooge, 1957) is an empirical method using watershed area and rainfall intensity as variables. Since that time numerous formulas have been proposed to predict peak discharge rates from rainfall intensities, for example, Gregory and Arnold (1932), Krimgold (1938), and Roe and Snyder (1943). These formulas take the general form

$$Q = K I^a A^b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where Q is the peak discharge rate, I is the rainfall intensity, A is the area of the watershed, and K, a, and b are empirical constants. The formulas are not considered valid unless the duration of the storm is at least equal to the travel time of the watershed.

MULTIPLE CORRELATION METHODS

During the past several decades hydrologists have shown that the total depth of rainfall (excess rainfall) which appears as the surface runoff part of the hydrograph may be estimated for a given watershed from a multiple correlation using such variables as antecedent precipitation index, season of year, and storm duration and intensity. The time distribution of discharge resulting from this excess rain has then been determined by unit hydrograph methods, or by a graphical correlation of the amount of excess rainfall during a short time increment vs runoff rate during the increment (Kohler and Linsley, 1951; Kresge and Nordenson, 1955; Miller and Paulhus, 1957; and Paulhus and Miller, 1957).

Unit Hydrograph: Since the physical characteristics of a basin--shape, size, slope, etc.--are constant, the hydrographs from storms of similar characteristics might be expected to have considerable similarity in shape. This concept is the basis of the unit hydrograph. The unit hydrograph method, proposed by Sherman (1932), yields a "typical" hydrograph for the watershed.

The unit hydrograph is defined as the hydrograph of one inch direct runoff (total runoff minus base flow) from a storm of specified duration. It is obtained by adjusting the ordinates of the hydrograph resulting from a storm of the specified duration so that the volume of direct runoff under the hydrograph is one inch. This is usually done for several storms of similar duration, and the hydrographs are superimposed to give a typical (average) unit hydrograph. Theoretically, a separate unit hydrograph is required for each duration of rainfall; however, unit

hydrographs for various durations can be constructed from a unit hydrograph of a single duration (Linsley, Kohler, and Paulhus, 1958).

Synthetic unit hydrograph: Often on small watersheds insufficient streamflow records are available from which to construct unit hydrographs. In this case the synthetic unit hydrograph method introduced by Snyder (1938) may be used. In this method the time lag, usually defined as the time interval between the centroid of rainfall and the hydrograph peak, is determined as an empirical function of the length and slope of the stream channel. The unit hydrograph peak for a given rainfall duration is then expressed as a function of drainage area and time lag. The hydrograph is assigned a shape from another empirical relation. The unit hydrograph thus synthesized can be applied by the same techniques as ordinary unit hydrographs to predict the peak discharge from a given rainfall excess.

Synthetic unit hydrographs have been applied specifically to small watersheds by Hickock, Keppel, and Rafferty (1959). They point out that hydrograph synthesis can only be applied on watersheds where major floods result from single storms producing runoff from the entire watershed. In the Southwest, this limits the use of synthetic hydrographs to watersheds of less than 1000 acres. The limit for eastern Montana is expected to be of the same order of magnitude. Renard and Keppel (1966) have developed a special technique for using synthetic unit hydrographs on larger ephemeral streams in the Southwest. In this technique, unit hydrographs are synthesized for portions of the watershed and routed to the mouth accounting for transmission losses.

For watersheds larger than a few square miles in area, the labor required to apply this method is very great. The labor involved may be reduced somewhat by the use of the electronic computer.

Multivariate methods: Lewis (1968) (in progress) has applied multivariate analysis to the four watersheds gaged by the Drainage Correlation Research Project in order to obtain a multiple correlation applicable to eastern Montana watersheds. Lewis's method relates peak discharge rates to watershed-related variables and meteorological variables in a single correlation. His results are not available at this writing.

Multiple correlation methods can be used to estimate peak floods for long recurrence intervals for watersheds with short streamflow records. They are, however, comparatively laborious to use.

FREQUENCY ANALYSIS

Frequency analysis is a technique well adapted to use on small watersheds. The major difficulty in the use of frequency analysis to determine the magnitude of floods from streamflow data is the shortage of long records. Harrold (1945) used streamflow records of less than ten years' length to predict floods of various recurrence intervals. He adjusted the data for non-representativeness of sample by use of rainfall frequency data. More recently Chow (1958) has suggested that synthetic peak discharges with corresponding recurrence intervals obtained by multiple correlation methods (as described in the preceding section) may be used to extend short records on small watersheds in

frequency analysis.

The method of frequency analysis treats hydrologic data as a statistical variable. The frequency distribution of the data is examined by an analytical approach and the magnitudes of the variable for various recurrence intervals are determined. The recurrence interval is defined as the average interval of time within which a given magnitude of the variable is equaled or exceeded (Andrews, 1957).

Figure (1-a) shows a set of recorded hydrologic data. Only the portion of the data of large magnitude is significant in predicting extreme events. Two methods are used to exclude the smaller portion of the data from the analysis. One method, the annual maximum series, uses the series of single largest values recorded in each year. The other, the partial-duration or exceedence series, uses the series of values above a selected base value irrespective of time of occurrence. The series of exceedence values is more difficult to study statistically, and Chow (1951) has shown that results from its use do not differ significantly from those obtained from the annual series for recurrence intervals longer than ten years. For these reasons, the annual series will be used for the frequency analyses in this study.

The annual maximum series for the data of Figure (1-a) is shown in Figure (1-b). The recurrence interval for the data is computed from a plotting position formula. The generally recommended formula is

$$T = \frac{N + 1}{m} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

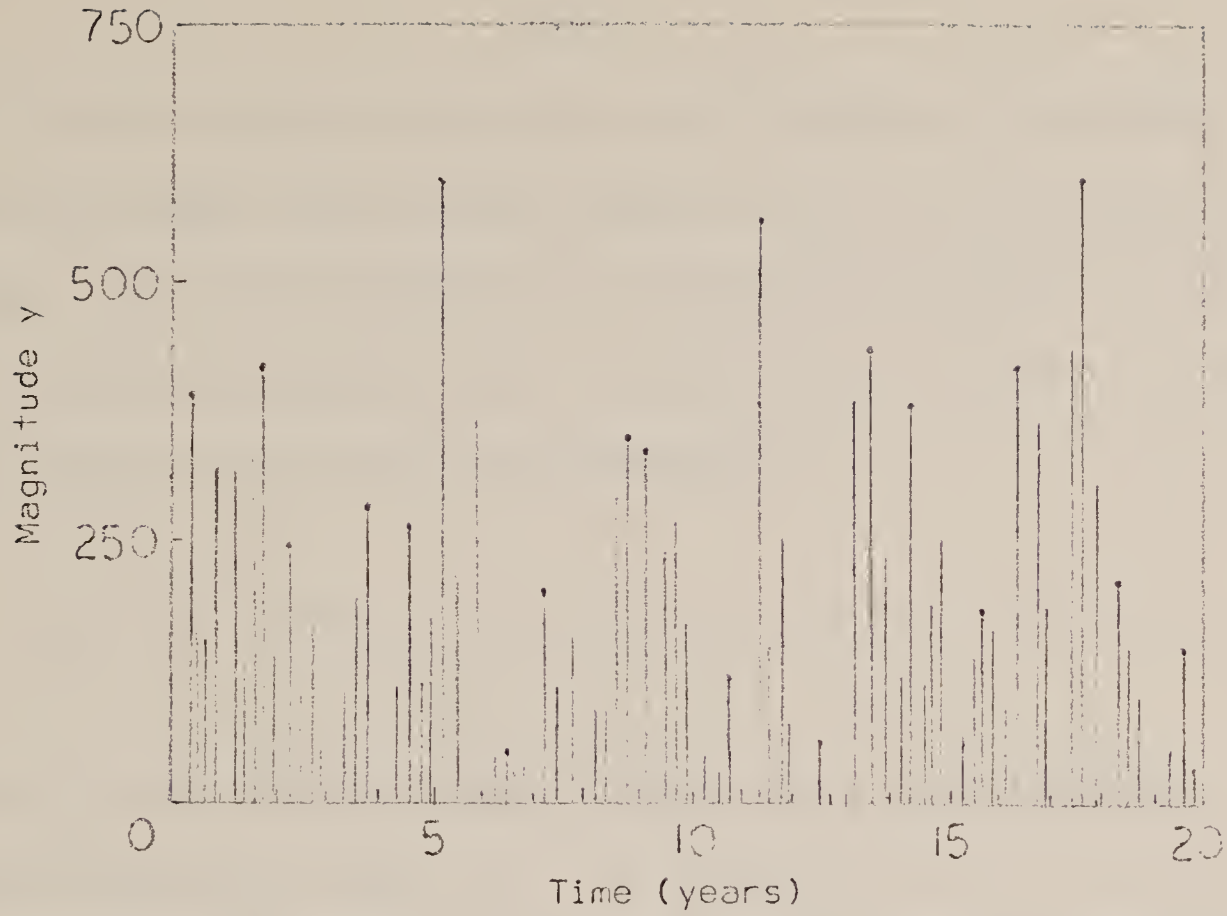


Figure (I-a): Original hydrologic data used to illustrate frequency analysis of hydrologic data.

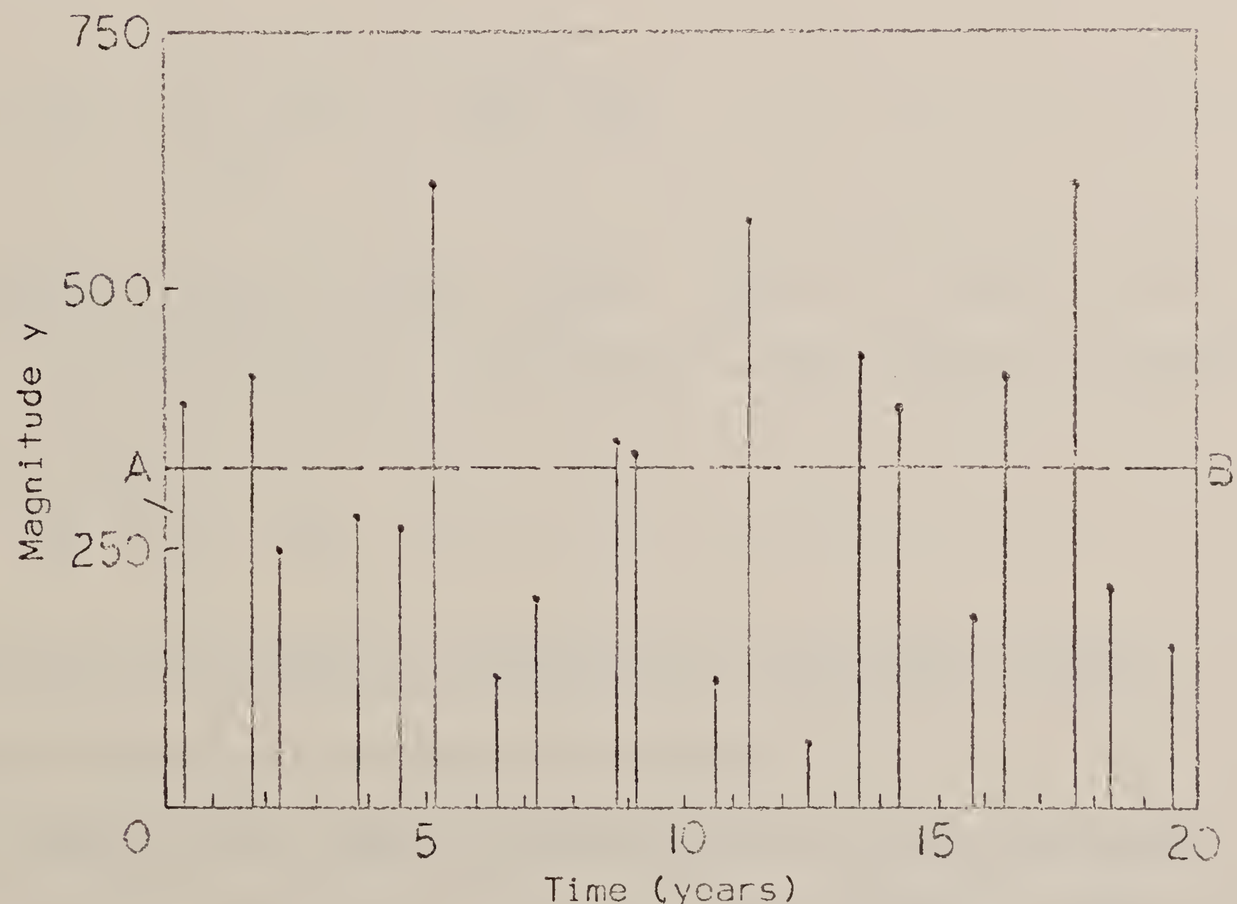


Figure (I-b): Annual maxima for the original data of Figure (I-a).

in which T is the recurrence interval, m is the rank of the values arranged in an order of decreasing magnitude, and N is the number of data values. Line AB in Figure (1-b) indicates the mean, $\bar{y}$, of the observed data whose magnitudes are represented by a variable, y. Any particular value of y can then be expressed by

$$y_i = \bar{y} + \Delta y_i \quad \dots\dots\dots\dots\dots\dots\dots\dots (3)$$

in which Δy_i is the departure from the mean and i is a subscript denoting the particular value of y. By statistics it has been shown that Δy_i can be expressed by the product of the standard deviation, σ, defined as,

$$\sigma = \sum_{j=1}^{n} [(y_j - \bar{y})^2] / n \quad \dots\dots\dots\dots\dots (4)$$

and a frequency factor, K (Chow, 1958). Values of K depend on the frequency distribution used, and increase as the recurrence interval increases. Thus,

$$y_i = \bar{y} + \sigma K \quad \dots\dots\dots\dots\dots\dots\dots\dots (5)$$

The selection of a frequency distribution is arbitrary and any distribution which fits the data can be used.

The extreme value (Gumbel) method is easy to apply and gives comparable answers to several commonly used distributions (Huff and Neill, 1959; Majumdar and Sawhney, 1965).

The extreme value distribution is obtained from the following assumptions (Gumbel, 1958):

1) The data is divided into N samples each of size n.

2) The distribution becomes defined as N and n become infinite.

3) A finite set of data can be fitted to the distribution by the method of least squares.

To apply the distribution to hydrologic analysis the following additional assumptions must be made:

1) The number of recorded values during one year is large enough to be considered infinite.

2) The distribution of the original data is of the exponential type.

3) The sample size is constant.

It is clear that these last assumptions are never exactly met. Experience has shown, however, that the fit is usually better than might be expected from these considerations.

Chow (1958) has derived the value of K for the extreme value (Gumbel) distribution as

$$K = \frac{1}{\sigma_n} (\ln T - \bar{y}_n) \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

in which σ_n and $\bar{y}_n$ are the reduced standard deviation and reduced mean and depend only on sample size (years of record) (tabulated in Gumbel, 1958), and T, as before, is the recurrence interval.

(ln T is the logarithm of T taken to the Naperian base e).

The log-normal distribution (log-probability law) is another frequency distribution commonly used in hydrology. It is based on the assumption that the logarithm of the variable, y, follows a normal distribution. Use of the log-normal distribution is equivalent to applying a logarithmic transformation to the data and fitting the transformed data to a normal distribution.

Chow (1958) has shown that the frequency factor, K, for the log-normal distribution is given by

$$K = \frac{e^{(\sigma K_y - \sigma/2)} - 1}{(e^{\sigma^2} - 1)^{\frac{1}{2}}} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

where $K_y = (y_i - \bar{y})/\sigma$. The other quantities are defined as above. For computational purposes, K has been tabulated as a function of the coefficient of variation, C_v, for given recurrence intervals (see Appendix A). The coefficient of variation is defined as

$$C_v = \sigma/\bar{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Application of the log-normal distribution to hydrologic analysis is based on the following assumptions (Chow, 1958):

1) The hydrologic event is the result of the joint action of many causative factors.

2) The variable y is equal to the product of a large number of r

independent magnitudes, x_1, x_2, . . . x_r, which are due to the r causative factors.

Figure (1-c) and Figure (1-d) show theoretical recurrence distributions which fit the data in Figure (1-b). It is usually convenient, however, to transform the recurrence scale so that the relation plots as a straight line. Special graph paper is available with transformed recurrence scales to fit common recurrence distributions. Figure (1-e) shows the data fitted to an extreme value distribution and Figure (1-f) shows the data fitted to a log-normal distribution.

Confidence curves can be constructed to estimate how accurately the theoretical distribution fits the data (as shown, for example, in Figure (1-e)). According to a method given by Gumbel (1958), confidence curves for the extreme value distribution are described by the equation

$$y_{cc} = y \pm (r.s.e) \ (\sigma) \ /(\sqrt{n} \ \sigma_n) \quad . . . . (9)$$

where y_{cc} is the ordinate of the confidence curve, y is the corresponding ordinate of the recurrence line, r.s.e is the "reduced standard error," a statistic computed and tabulated by Gumbel for various values of n and σ. The values of n and σ_n are defined as above. These confidence curves show the range of recurrence intervals within which a given magnitude of the variable may be expected with a probability of approximately 2/3. Equation (9) applies only for analysis according to the extreme value

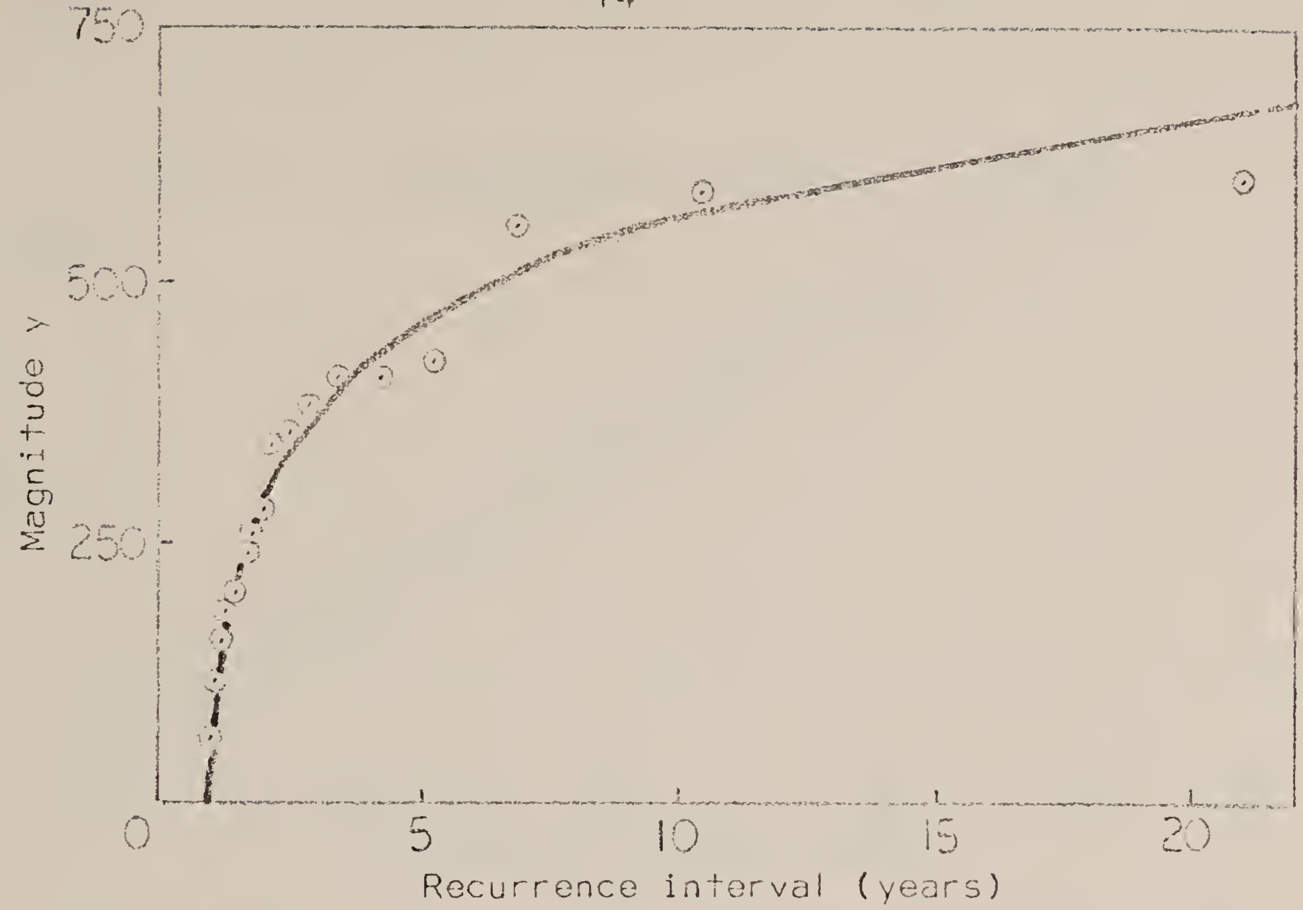


Figure (1-c): Data of Figure (1-b) fitted to an extreme value distribution, plotted on arithmetic scale.

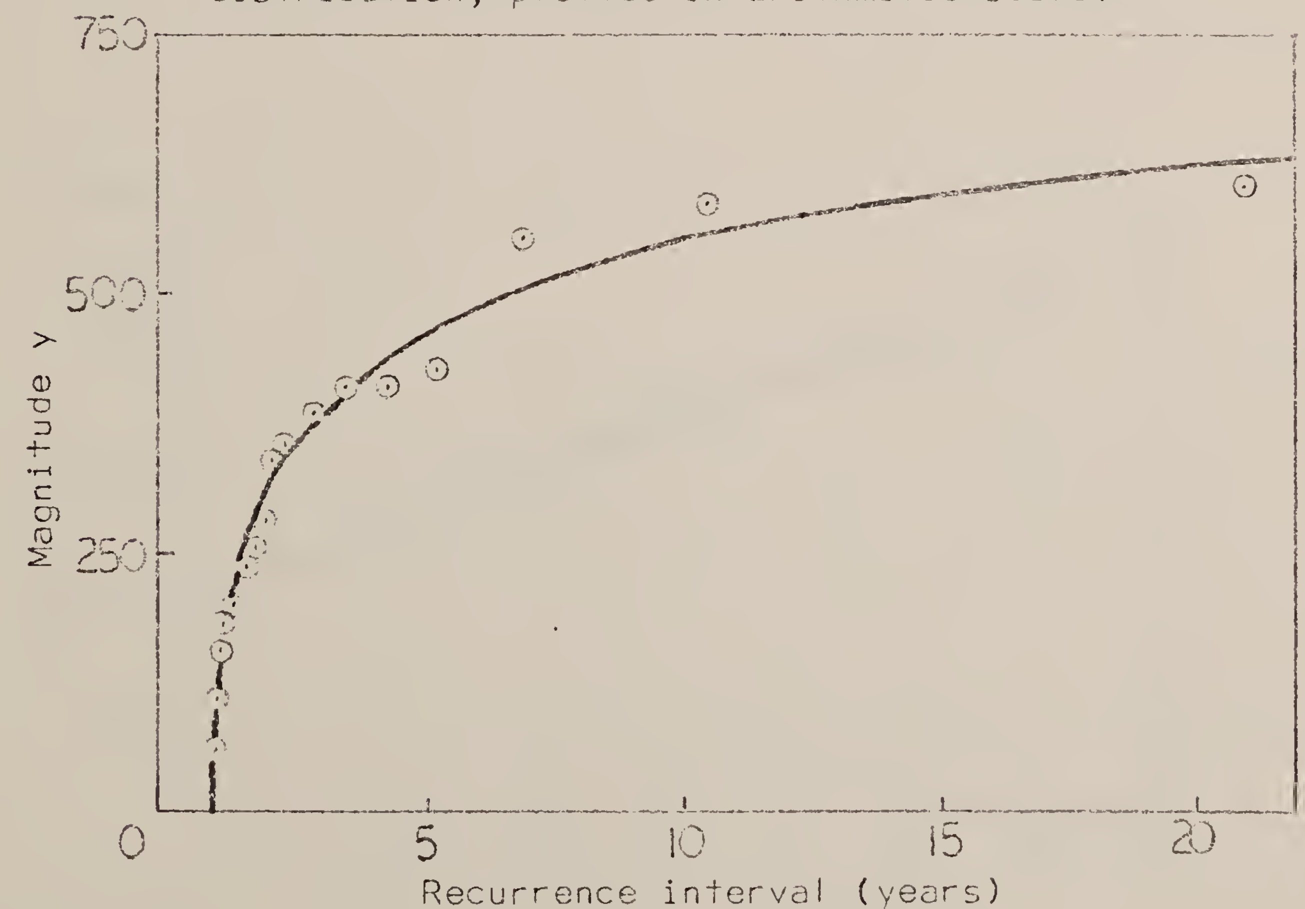


Figure (1-d): Data of Figure (1-b) fitted to a log-normal distribution, plotted on arithmetic scale.

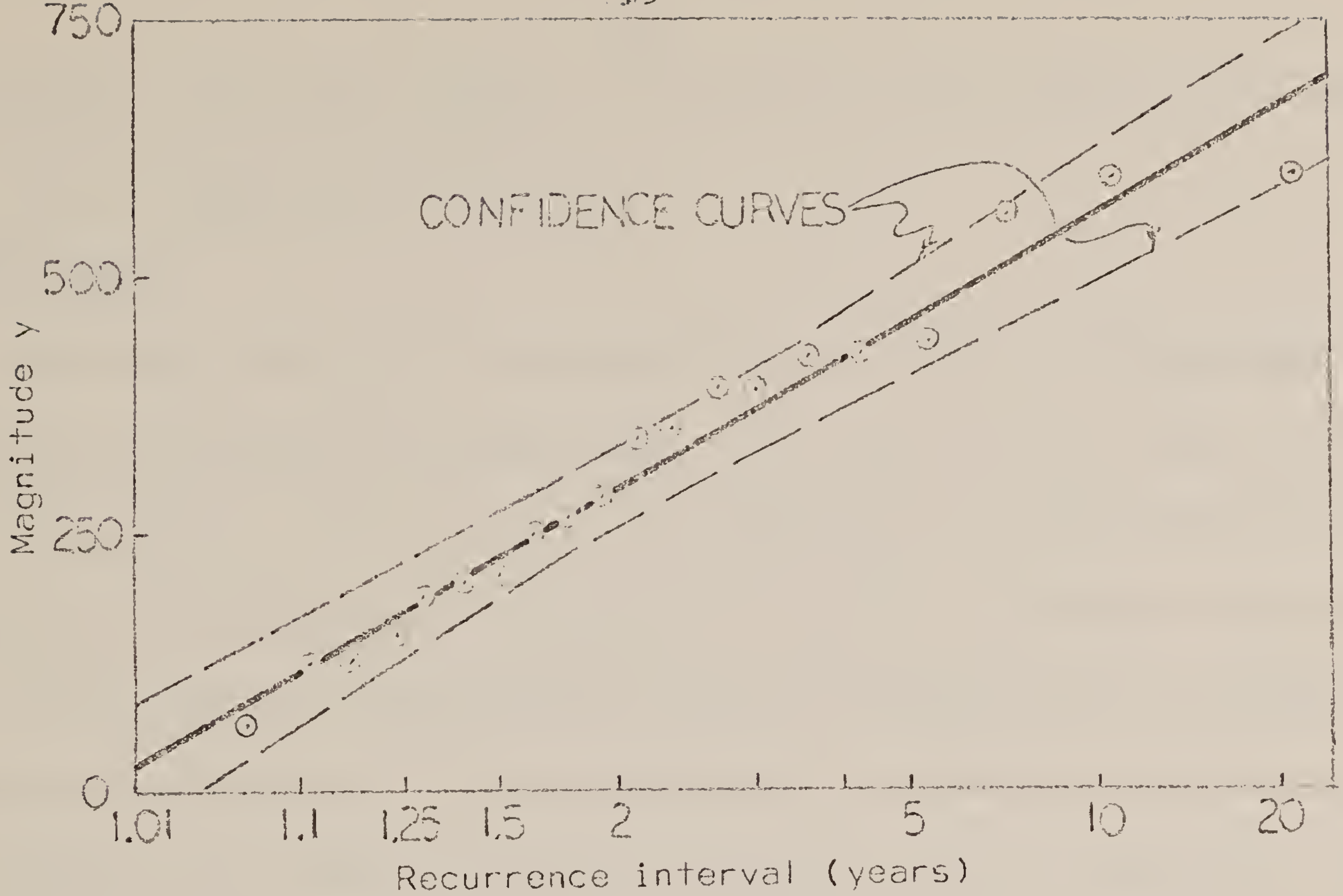


Figure (I-e): Data of Figure (I-b) fitted to an extreme value distribution, plotted on extreme value paper.

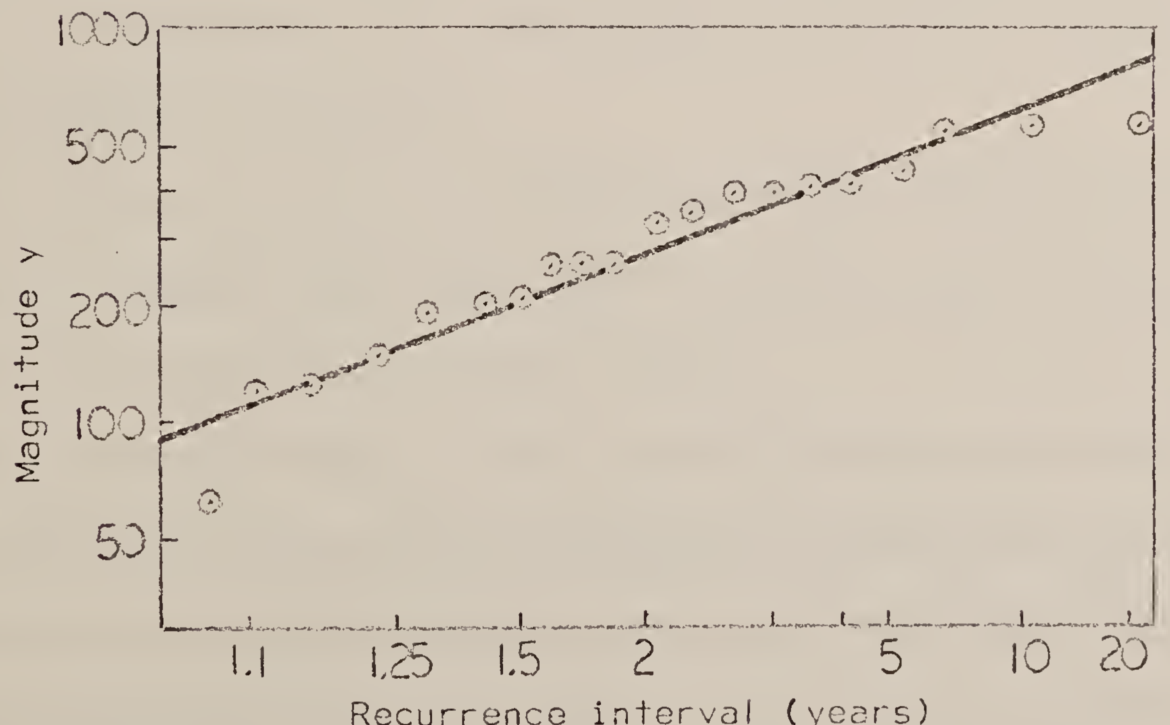


Figure (I-f): Data of Figure (I-b) fitted to a log-normal distribution, plotted on log-normal paper.

distribution, but within ordinary ranges of C_v it is approximately valid also for the log-normal distribution.

GRADEX METHOD

The Gradex method of flood prediction, developed for Electricite de France by Guillot and Duband (1967), has been used for design problems for several years. It is based on the hypothesis that the frequency of rainfall and the frequency of floods are parallel at extreme conditions (i.e. on an extreme value frequency plot the distribution of rainfall is asymptotically parallel to the distribution of extreme flood discharges). The inverse of the slope of these two parallel lines is termed the Gradex and may be obtained for certain watersheds from a few years of observations of daily rainfall. Some of the assumptions necessary in the development of the Gradex concept are

1) The frequency of the local daily precipitation has an exponential rate of depletion.

2) In flood conditions, the soil approaches saturation and each increment of rainfall, dR, gives rise to a nearly equal increase of daily discharge, Q; dQ tends to be equal to dR.

For watersheds ranging in size from 40 to 4000 square miles the authors found that using daily precipitation values give satisfactory predictions of mean daily discharge rates, but they imply that for other watersheds the proper time unit might be a few hours or several days.

SOIL CONSERVATION SERVICE METHOD

The U.S. Department of Agriculture, Soil Conservation Service (1964) uses a procedure adapted mainly for reservoir design from which floods of various recurrence intervals can be predicted. The watershed is divided into reaches for which hydrologic soil groups, cover types, and land uses are inventoried. Hydrologic soil-cover complexes are assigned on the basis of the inventory. The soil-cover complexes are used in an empirical relation to estimate the volume of runoff from historical storms, or storms of various frequency obtained from Hershfield (1961). The recurrence relation of volume of runoff is established from these data. Peak flow rate is obtained by routing events of desired runoff volume through the various reaches of the watershed by unit hydrograph techniques. A computer program is available which performs the computations for the Soil Conservation Service method.

Ferris (1968) has applied the SCS method to a single event on Duck Creek in Prairie and McCone Counties, Montana. He found the volume of runoff predicted by the SCS method to compare favorably with the runoff volume actually recorded; but the peak discharge predicted by the SCS method was more than twice the discharge actually obtained. Ferris believes the discrepancy to be primarily due to the difficulty in selecting proper values for the watershed parameters.

PREDICTION OF FLOODS ON SMALL MONTANA WATERSHEDS

Dalrymple (1960) developed a method to determine magnitude and

frequency of floods. His method is accepted as a standard method by the U.S. Department of Interior Geological Survey and is used by their offices throughout the United States. His analysis provides two curves. The first curve expresses the ratio of the peak flow to the mean annual flood as a function of the recurrence interval for a homogeneous hydrologic region. The second curve relates the mean annual flood to drainage area and to other basin characteristics, if needed. Applications of this method have been made by Berwick (1958), Boner (1963), and Patterson (1966) for areas in Montana.

Berwick (1958) developed curves applicable to most of Montana east of the 109th meridian (a north-south line a few miles east of Lewistown and Columbus) which can be used to predict floods with recurrence intervals shorter than 20 years. Berwick's results have been revised and extended by Boner (1963). According to Boner, the mean annual flood, $\bar{Q}_T$, is estimated from 40 gaging stations by the formula:

$$\bar{Q}_T = 278,000 \ A^{0.382} \ E^{-1.255} \ L^{0.433} \ G \quad \ldots\ldots \ (10)$$

where A is the drainage area in square miles, E is the average of the elevations at 2/10 and 8/10 meander length of the main stream course in feet, L is the meander length of the main stream course in miles, and G is a dimensionless geographical factor. The standard error of estimate in applying this formula to the 40 watersheds studied ranges between -27 and +37 percent of the mean. Part of the standard error is attributed to errors in the frequency curves used in developing the

formula. A composite frequency curve expressed as the ratio of the extreme flood to the mean annual flood is given for recurrence intervals up to 25 years. Boner states that his curves should be applied only to watersheds from 0.5 to 3200 square miles in area.

Patterson (1966) has developed composite frequency curves applicable for most of the Missouri River drainage above Sioux City, Iowa. Curves are given for recurrence intervals as long as 50 years. Multiple correlations relate the mean annual flood to watershed area and to elevation in some mountainous areas. Patterson suggests that his method is applicable to watersheds of 15 to 10,000 square miles in area. Many of the small streams used to develop the curves for some localities are at high altitudes, where snow melt runoff is predominant. For these localities the minimum area for which the curves can be applied may be as much as 100 square miles.

Boner and Omang (1967) have developed a method for predicting floods on Montana watersheds less than 100 square miles in area. They divide Montana into thirteen regions for which the ratio of the 25-year flood to the 10-year flood is given as listed in Table (1). For regions A, B, C, D, E, and F the 10-year flood is related to the drainage area and the average annual runoff of the watershed. The average annual runoff is obtained from an isoplethal map in the report. For the other regions, the 10-year flood is related to the drainage area only. Using the relations given, the 10-year and 25-year floods can be estimated. Floods of recurrence intervals other than 10 or 25 years cannot be predicted.

Table (1): Ratios of the 25-year flood to the 10-year flood. (After Boner and Omang, 1967)

Region	Counties comprising major portions of the region	Median ratio
A	Lincoln	1.3
B	Flathead	1.2
C	Flathead, Missoula, Powell	1.3
D	Sanders, Missoula, Granite	1.25
E	Glacier	1.25
F	Glacier, Toole, Liberty, Teton, Pondera	1.5
G	Broadwater, Judith Basin, Meagher, Wheatland	1.4
H	Beaverhead, Silver Bow, Madison	1.25
I	Gallatin	1.25
J	Park, Sweetgrass, Stillwater, Carbon	1.2
K	Hill, Blaine, Phillips, Valley, Daniels, Roosevelt, Sheridan	1.45
L	Golden Valley, Musselshell, Fergus, Petroleum, Garfield, McCone	1.45
M	Yellowstone, Treasure, Rosebud, Big Horn, Custer, Powder River, Prairie, Fallon, Dawson, Carter	1.45

Reliability of the 10-year flood is fair, and reliability of the 25-year flood is poor, according to Boner and Omang.

CHAPTER III

DERIVATION OF THE RAINFALL-DISCHARGE INDUCTION EQUATION

This investigation uses a theoretically based equation for the prediction of a peak discharge rate for a desired recurrence interval. This chapter presents the derivation of this equation. The equation to be derived uses the mean annual peak discharge rate of the stream (which can be estimated fairly accurately from short streamflow records) together with three theoretical factors which are termed the rainfall intensity ratio, R, the rainfall-discharge recurrence factor, D, and the rain-snow-base flow interaction ratio, F. R is defined as the ratio of the i-year rainfall intensity of given duration to the mean annual rainfall intensity of the same duration where i is a specified recurrence interval. D expresses the relationship between the rainfall recurrence relation for specified duration and the recurrence relation of peak annual discharges. F reflects the relative dependence or independence of the frequency curves of rainfall-induced flows and snow-melt induced flows.

When a rainstorm occurs on a watershed, the water reaching the ground surface may be dispersed in several ways. Some of the water may infiltrate into the soil, where it will be eventually evaporated, transpired by plants, or enter ground water. Some water may be evaporated immediately into the atmosphere, and some may be retained in ground depressions as puddles. The water that is not dispersed in one of these ways must flow over the ground surface under the influence of gravity to enter a watercourse as direct runoff.

If the rate at which the rain falls on the watershed (rainfall intensity) is less than the sum of the infiltration, evaporation, and transpiration rates no direct runoff will result. If the rainfall intensity exceeds the sum of these rates for a long enough time to fill surface depressions streamflow will result. The increment of rainfall which contributes to direct runoff is termed the rainfall excess.

On a given watershed, when the rainfall intensity is sufficient to cause runoff, the characteristics of the hydrograph depend on duration, uniformity (within area), steadiness (in time) of the rainfall, and the antecedent condition of the watershed. The duration effect may be divided into two cases.

1) Long durations during which the streamflow will reach a steady state condition if the rainfall intensity is steady and uniform.

2) Shorter durations during which the streamflow will not reach a steady state condition.

The simplest case to analyze, however, is a steady, uniform rainfall falling on the watershed for a long period of time.

If a rainfall excess with a steady intensity and of long duration should occur uniformly over an entire watershed, the streamflow at the mouth of the watershed would gradually increase until water from the farthest point on the watershed had sufficient time to reach the mouth of the watershed. The time required for the water to reach the mouth of the watershed from the most remote point is defined as the travel time of

the watershed. The streamflow would continue at this rate (a steady state condition) until the rainfall excess rate changed.

For this case, the maximum rate of streamflow (in excess of base flow) at the steady state condition is related to the rainfall intensity and the infiltration rate by an equation of continuity.

$$Q_R = IA - SA \qquad \text{. . . . . (11)}$$

where Q_R is the peak rate of discharge due to rainfall in cubic feet per second (cfs), I is the rainfall intensity in inches per hour (in./hr), S is the infiltration rate in in./hr, and A is the area of the watershed in acres. (One cfs is numerically equivalent, within one percent, to one acre inch per hour.) At steady state, surface retention and channel storage are constant and do not enter into Equation (11).

Ordinarily, rainstorms are not of steady, uniform intensity, and many storms are of insufficient duration for a steady state streamflow condition to occur. In order to develop a relation applying to these cases, consider a series of rainstorms which occur on a watershed, each having the same average intensity, and of the same duration, but with intensity varying across the watershed (non-uniform) and varying with time (unsteady). For example, such a series might appear as listed in Table (2) for a hypothetical watershed of 8820 acres area. Because the antecedent conditions (and hence infiltration rate) are different for each storm, and because each storm has a different rainfall intensity distribution each will produce a different peak discharge rate. For the

Table (2): A hypothetical series of rainstorms on a 13.8 square mile (8820 acre) watershed.

Storm	Antecedent condition	Average intensity (in./hr)	Duration (hrs)	Infiltration rate (in./hr)	Peak discharge rate (cfs)
A	average	1.0	12	0.6	5800
B	dry	1.0	12	0.6	4800
C	moist	1.0	12	0.1	4600
D	average	1.0	12	0.4	5300
E	dry	1.0	12	0.7	3700
F	moist	1.0	12	0.4	4400
G	average	1.0	12	0.4	4800
H	average	1.0	12	0.4	4800
I	average	1.0	12	0.5	5100
J	dry	1.0	12	0.6	4300
K	moist	1.0	12	0.2	6600
L	dry	1.0	12	0.8	2800
M	dry	1.0	12	0.7	3300
N	moist	1.0	12	0.2	6200
O	dry	1.0	12	0.6	4400
P	moist	1.0	12	0.3	4800
Q	moist	1.0	12	0.3	5400
R	average	1.0	12	0.5	5300
S	average	1.0	12	0.4	5600
T	moist	1.0	12	0.3	5300
Mean:		1.0	---	0.45	4860

data of Table (2), it is seen that a rainstorm with average intensity 1.0 in./hr and 12-hour duration produces, on the average, a peak discharge of 4860 cfs and has an infiltration rate of 0.45 in./hr associated with it.

Instead of considering only storms of a single intensity, the population of all runoff events caused by rainfalls of the same duration on the watershed may be considered. Each runoff event will have an average rainfall intensity, an average infiltration rate, and a peak discharge rate associated with it. A theoretical frequency distribution may be fitted to the largest values of rainfall intensity for that storm duration during each year of record. For the watershed of Table (2) a theoretical rainfall-intensity frequency distribution might appear as shown in Figure (2). From Figure (2) it is evident that a 12-hour duration rainfall with intensity of 1.0 in./hr has a recurrence interval of 10 years associated with it. In a similar manner a theoretical frequency distribution may be fitted to the smallest infiltration rate observed during each year of record, as shown in Figure (2). From Figure (2) it is seen that an infiltration rate of 0.45 in./hr has a recurrence interval of 11 years associated with it. A frequency distribution may be fitted to peak annual discharges in a like manner, such as shown in Figure (2), from which it is seen that a peak discharge of 4860 cfs has a recurrence interval of 15 years associated with it. The maximum average rainfall intensity, minimum infiltration rate, and maximum discharge rate observed during a given year may or may not all be associated with the same runoff event. For the above example, it is seen that an 11-year

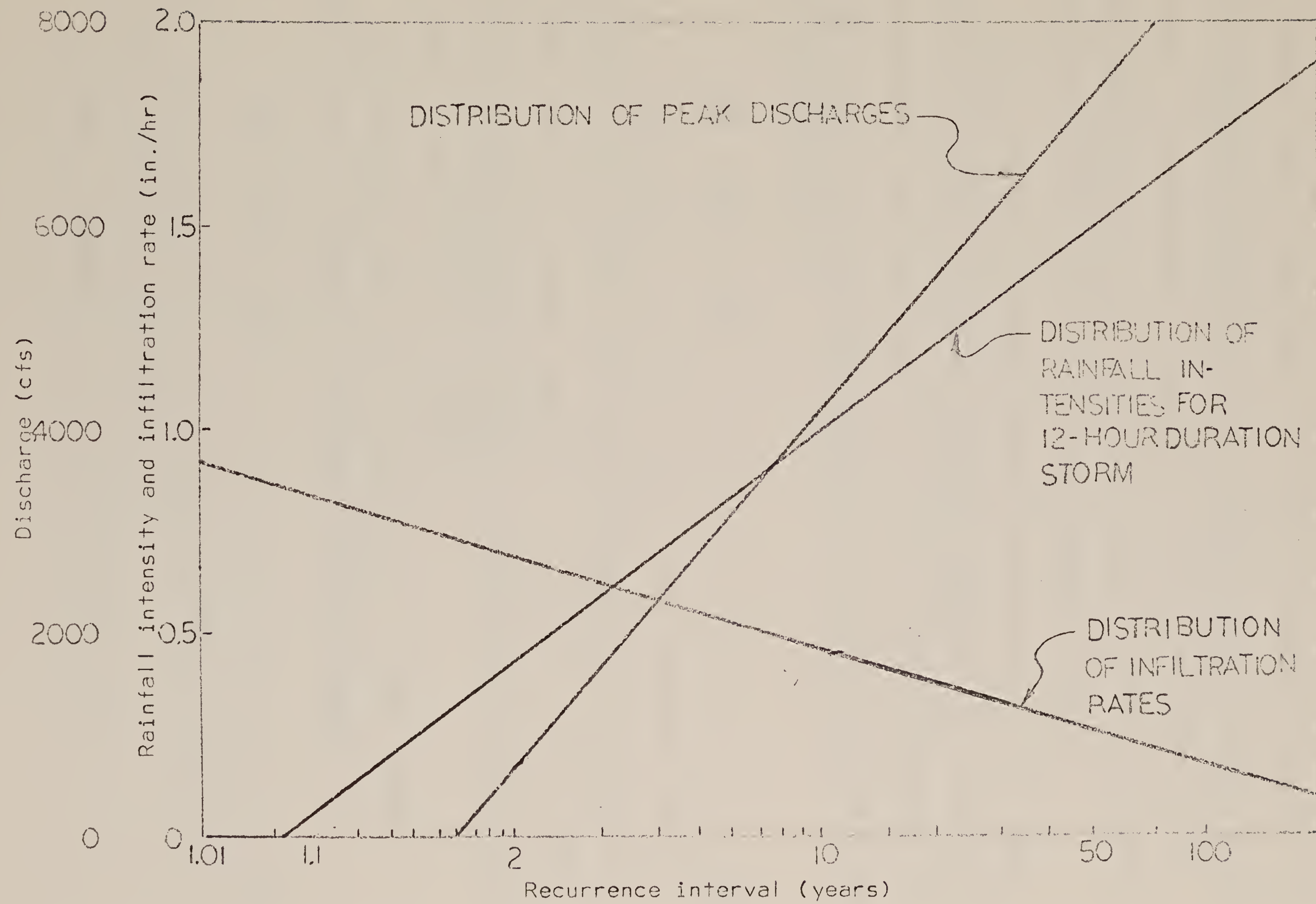


Figure (2): Theoretical frequency distributions for the hypothetical watershed of Table (2).

infiltration rate in conjunction with a 10-year rainfall intensity (for 12-hour duration storms) will, on the average, produce a 15-year peak discharge. This phenomenon is expected since the combination of the proper infiltration rate and rainfall intensity to produce the 15-year flood will occur more rarely than either individual factor.

Using recurrence intervals assigned in this manner, it is, then, reasonable to write an equation analogous to Equation (11) to relate the average value of peak discharge to the average rainfall intensity (for storms of given duration) which produces it and the average infiltration rate associated with it.

$$Q_R^i = I^j A - S^k A \qquad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

where the superscripts i, j, and k represent the recurrence intervals assigned to each quantity. For the above example i is 15 years, j is 10 years, and k is 11 years, thus 4860 = (1.0) (8820) - (0.45) (8820). Note, for a series of storms of another duration (say 24 hours) the value of j in Equation (12) would be different.

For the special case where the discharge is the mean annual discharge due to rainfall, $\bar{Q}_R$, Equation (12) may be written

$$\bar{Q}_R = I^p A - S^q A \qquad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where p and q designate the corresponding recurrence intervals for rainfall intensity and infiltration rate.

The ratio of Equation (12) to Equation (13) is, then,

$$\frac{Q_R^i}{\bar{Q}_R} = \frac{I^j - S^k}{I^p - S^q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

or, after rearranging,

$$\frac{Q_R^i}{\bar{Q}_R} = \frac{I^i}{\bar{I}} \left[\frac{\bar{I}}{I^p} \; \frac{I^j}{I^i} \left(\frac{1 - \dfrac{S^k}{I^j}}{1 - \dfrac{S^q}{I^p}} \right) \right] \quad \ldots\ldots\ldots\ldots \quad (15)$$

If the expression in brackets is denoted by D, then

$$\frac{Q_R^i}{\bar{Q}_R} = \frac{I^i}{\bar{I}} \, D \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where,

$$D = \frac{\bar{I}}{I^p} \; \frac{I^j}{I^i} \left(\frac{1 - \dfrac{S^k}{I^j}}{1 - \dfrac{S^q}{I^p}} \right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Equation (16) is an expression relating Q and I which may be used with any frequency distribution (e.g. extreme value or log-normal). In Equation (17) the ratio $\bar{I}/I^p$ reflects the difference between the mean annual peak rainfall intensity and the intensity which produces the mean annual flood. It always has a value less than 1.0. I^j/I^i reflects the difference in the i-year intensity, and the intensity which produces

the i-year flood. It is always larger than 1.0, but approaches 1.0 when i and j are large. The parenthesized term reflects the difference in the proportion of rainfall contributing to infiltration for common and rare events and is greater than 1.0. D, then, is a rainfall-discharge recurrence factor, reflecting the differences between the distribution of discharges and the distribution of rainfall intensities.

If it is assumed that both rainfall intensities and peak discharge rates can be fitted by the first asymptotic extreme value distribution as proposed by Gumbel (1958), then Q_R^i and I^i are expressed by

$$Q_R^i = \bar{Q}_R + \frac{\sigma_Q}{\sigma_{nQ}} (\ln i - \bar{y}_{nQ}) \qquad \ldots\ldots\ldots\ldots (18)$$

$$I^i = \bar{I} + \frac{\sigma_I}{\sigma_{nI}} (\ln i - \bar{y}_{nI}) \qquad \ldots\ldots\ldots\ldots (19)$$

where σ_Q is the standard deviation of the discharges, σ_I is the standard deviation of the rainfall intensities. The reduced mean and standard deviations, $\bar{y}_{nQ}$, σ_{nQ}, $\bar{y}_{nI}$, and σ_{nI} are parameters which are functions only of the length of the discharge and rainfall records.

Taking the ratio of Q_R^i to I^i gives

$$\frac{Q_R^i}{I^i} = \frac{\bar{Q}_R + \frac{\sigma_Q}{\sigma_{nQ}} (\ln i - \bar{y}_{nQ})}{\bar{I} + \frac{\sigma_I}{\sigma_{nI}} (\ln i - \bar{y}_{nI})} \qquad \ldots\ldots\ldots\ldots (20)$$

or,

$$\frac{Q_R^i}{I^i} = \frac{\bar{Q}_R}{\bar{I}} \left[\frac{1 + \frac{\sigma_Q}{\bar{Q}} \frac{1}{\sigma_{nQ}} (\ln i - \bar{y}_{nQ})}{1 + \frac{\sigma_I}{\bar{I}} \frac{1}{\sigma_{nI}} (\ln i - \bar{y}_{nI})} \right] \quad \ldots \ldots (21)$$

or,

$$\frac{Q_R^i}{I^i} = \frac{\bar{Q}_R}{\bar{I}} \left[\frac{1 + C_{vQ} \frac{1}{\sigma_{nQ}} (\ln i - \bar{y}_{nQ})}{1 + C_{vI} \frac{1}{\sigma_{nQ}} (\ln i - \bar{y}_{nI})} \right] \quad \ldots \ldots (22)$$

where C_v is the coefficient of variation, (the standard deviation divided by the mean). Comparison of Equation (22) with Equation (16) gives a value of D based on the extreme value distribution function.

$$D = \frac{1 + C_{vQ} \frac{1}{\sigma_{nQ}} (\ln i - \bar{y}_{nQ})}{1 + C_{vI} \frac{1}{\sigma_{nI}} (\ln i - \bar{y}_{nI})} \quad \ldots \ldots (23)$$

The expressions for D given by Equation (17) and Equation (23) are closely related. C_{vQ} is a function of the average antecedent watershed conditions which determine S^k and S^q. C_{vI} reflects the relation between I^i, I^j, I^p, and $\bar{I}$.

The characteristics of D can be examined by considering the components of Equation (21). C_{vI} is a measure of the variability of rainfall intensities, and as such reflects meteorological characteristics

in the locality of the watershed. Chow (1952) has plotted isopleths which show that C_{vI} does not vary greatly within eastern Montana. C_{vQ} is a measure of the variability of discharges and thus reflects watershed characteristics. It is reasonable to assume that C_{vQ} does not vary greatly within a given area, where watershed characteristics are similar.

The same effects which cause C_{vI} to be larger or smaller in one location than in another are expected to cause an increase or decrease in C_{vQ}. Since C_{vQ} is in the numerator and C_{vI} is in the denominator of D, any change in the value of D will be smaller than that of C_{vI} or C_{vQ}.

Gumbel's values of $\bar{y}_{nQ}$, σ_{nQ}, $\bar{y}_{nI}$, and σ_{nI} are nearly constant over quite large ranges of record length and can be approximated to sufficient accuracy by representative values. For instance, $\bar{y}_{nQ}$ is 0.52 and σ_{nQ} is 1.06 for record length 20 years, while $\bar{y}_{nQ}$ is 0.55 and σ_{nQ} is 1.16 for record length 50 years.

If the total discharge with recurrence interval i including the effects of rainfall, snow melt, and base flow, is designated by Q_T^i, the rain-snow-base flow interaction ratio, F, can be defined to express the relation between Q_R^i and Q_T^i.

$$F = \frac{Q_R^i \ / \ \bar{Q}_R}{Q_T^i \ / \ \bar{Q}_T} \qquad \text{. . . . . (24)}$$

where $\bar{Q}_T$ is the mean annual discharge corresponding to Q_T^i. F reflects the interaction between the frequency curves of base flow, snow-melt-induced flow and rainfall-induced flow as they are combined to produce

the total-discharge frequency curve. In other words, it reflects the relative dependence or independence of the frequency curves at the mean and i-year levels. Because the interaction involved is complex, it is difficult to predict the characteristics of F; however, F may be evaluated empirically for watersheds for which the terms on the right hand side of Equation (24) are known. These empirical values of F may be used in Equation (24) to determine Q_T^i for other watersheds.

Rearranging Equation (24) gives

$$Q_T^i = \frac{Q_R^i}{\bar{Q}_R} \frac{\bar{Q}_T}{F} \qquad (25)$$

If the value of $Q_R^i/\bar{Q}_R$ is substituted from Equation (16), Equation (25) becomes,

$$Q_T^i = D \left(\frac{I^i}{\bar{I}}\right) \left(\frac{\bar{Q}_T}{F}\right) \qquad (26)$$

Let $I^i/\bar{I}$ be denoted by R, then,

$$Q_T^i = \left(\frac{DR}{F}\right) \bar{Q}_T \qquad (27)$$

Equation (27) relates the i-year flood to the mean annual flood by the three factors D, the rainfall-discharge recurrence factor; R, the rainfall-intensity ratio; and F, the rain-snow-base flow interaction ratio; which may be evaluated for any desired watershed.

For convenience in the analysis in this report, i will be taken as 50 years. Values of D, R, and F are determined for this case, and are not separately denoted. However, the values are different than they would be for other values of i.

CHAPTER IV

DATA ANALYSIS AND DETERMINATION OF COEFFICIENTS

In order to predict floods using Equation (25) the coefficients R, D, and F must be known. In order to evaluate these coefficients published rainfall data obtained from the U.S. Department of Commerce, Environmental Science Services Administration, and runoff data obtained from the U.S. Department of Interior Geological Survey were used. In this chapter the steps necessary in the determination of R, D, and F are described.

The coefficients R and D depend on the frequency distribution of rainfall intensities. The IBM 1620 and SDS Sigma 7 computers were used to scan the ESSA data in order to obtain pertinent rainfall values, and to construct frequency curves from these values.

The factor D also depends on the frequency distribution of rainfall-induced peak flows, as does F. In addition, F depends on the frequency distribution of total peak flows, and indirectly on the distribution of snow-melt-induced peak flows. Runoff data were analyzed in two separate phases. During the first phase techniques were developed and applied to several watersheds in one locality of Montana. During the second phase the techniques developed in the first phase were applied to other watersheds throughout Montana east of the continental divide.

Values of the coefficients R, D, and F were determined using the results of the frequency analyses of the rainfall and runoff data. The results of additional frequency analyses published in the literature were

also used where available. Isoplethal maps are presented from which values of R and D may be estimated for eastern Montana watersheds, and a representative value of F for eastern Montana is presented.

RAINFALL DATA ANALYSIS

Two types of rainfall data were used in this study, recording rain-gage data reported as amounts during clock hours, and non-recording rain-gage data, reported as amount per observational day[1]. The data were obtained from climatalogical data published by the Environmental Science Services Administration (1966, etc.) and punched-card decks of climatalogical data purchased from ESSA. A listing of stations used showing type and length of record is included in Table (3).

The hourly and daily precipitation data were analyzed on the IBM 1620 and SDS Sigma 7 computers using Fortran programs written by the author. One program determined the rainfall amount during consecutive blocks of time, e.g., from 12:00 M to 2:00 AM, from 2:00 AM to 4:00 AM, etc. for 1 hour, 2 hours, 3 hours, 4 hours, 6 hours, 8 hours, 12 hours, and 24 hours of rainfall. The program then selected and printed the maximum amounts during each year of record for each observational interval. The computer programs are listed in Appendix A. A graph showing the results for one station is shown in Figure (3). Although some of the

[1]Daily rainfall observations are made at the same time each day at individual stations; however, the time of observation varies widely from station to station.

Table (3): List of precipitation measuring stations whose records are used in the rainfall intensity-relation determination.

Station number[a]	Station name	Type of record[b]	Length of record[c] (years)
466	Barber	Non-recording	18
780	Big Timber	Non-recording	63
807	Billings WB AP	Recording[d]	20
877	Blackleaf	Non-recording	12
1088	Bredette	Recording	26
1127	Broadus	Recording	24
1169	Brockway	Non-recording	8
1342	Bynum 4SSE	Non-recording	12
1737	Choteau	Recording	28
1765	Circle 7N	Non-recording	19
1974	Conrad	Non-recording	49
2477	Dovetail 1N	Recording[d]	26
2571	Dupuyer	Recording[d]	13
2689	Ekalaka	Recording	20
2820	Ethridge	Non-recording	12
2996	Fishtail	Non-recording	12
3113	Fort Benton	Non-recording	20
3558	Glasgow WB AP	Recording[d]	20
3751	Great Falls WB AP	Recording[d]	20
3939	Harlowton	Non-recording	16
3996	Havre WB AP	Recording	44
4055	Helena WB AP	Recording	46
4358	Hysham	Non-recording	20
4538	Judith Gap	Non-recording	14
4558	Kalispell WB AP	Recording	37

[a]U.S. Weather Bureau designation.

[b]Present type of gage---earlier portion of record may be non-recording for recording stations.

[c]Only a portion of the Weather Bureau record, in some cases.

[d]Only 24-hour values were used in the analysis.

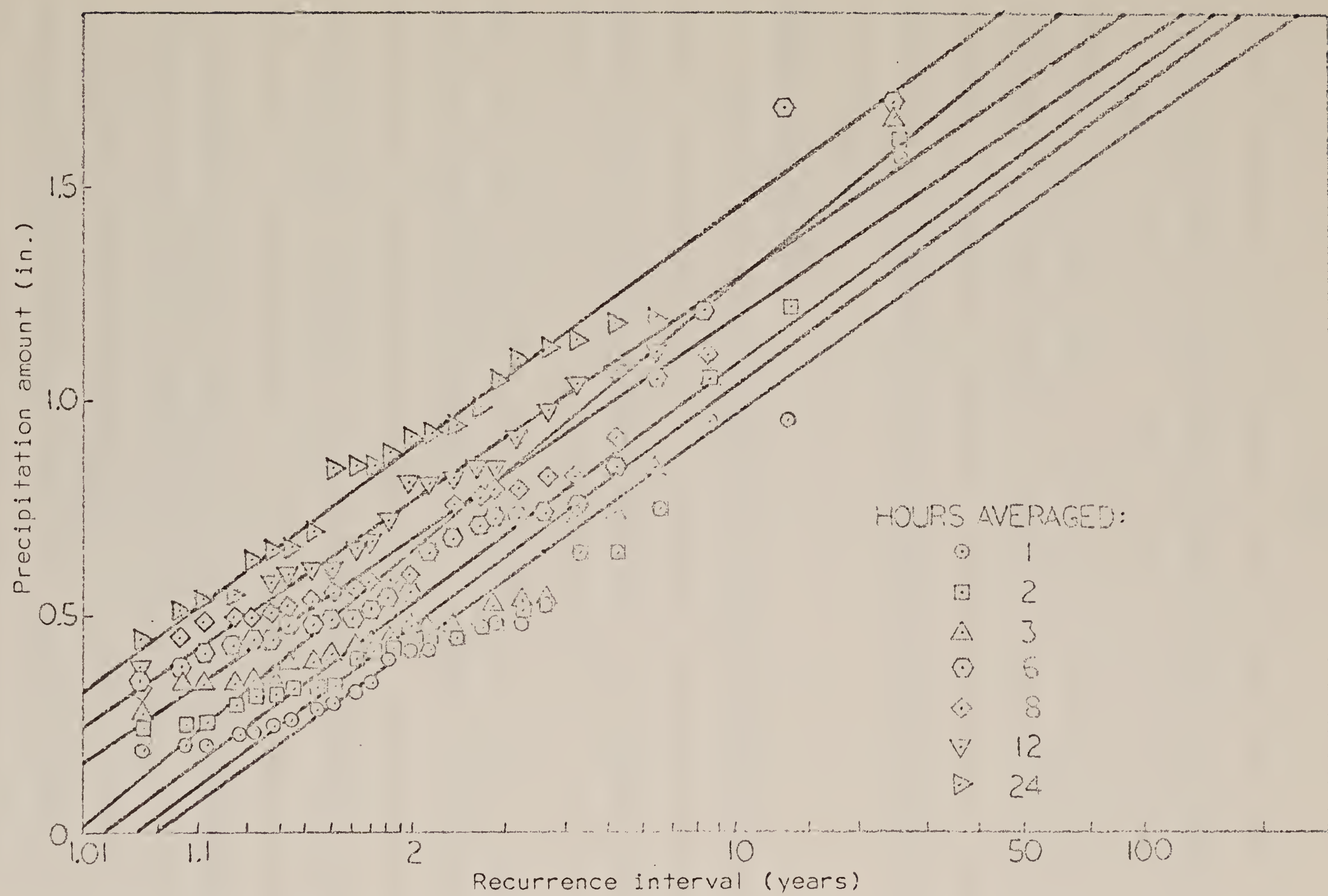
Table (3): List of precipitation measuring stations whose records are used in the rainfall intensity-relation determination (continued).

Station number[a]	Station name	Type of record[b]	Length of record[c] (years)
4663	Kings Hill	Recording	26
4985	Lewistown AP	Non-recording	20
5086	Livingston FAA AP	Recording	24
5235	Loweth	Non-recording	10
5337	Malta	Non-recording	20
5387	Martinsdale	Recording	25
5603	Melville 4W	Non-recording	7
5685	Miles City	Recording	19
5745	Missoula WB AP	Recording	16
6426	Pendroy	Non-recording	13
6918	Red Lodge	Non-recording	20
7214	Roundup	Non-recording	20
7560	Sidney	Non-recording	18
8169	Terry 25 NNW	Recording	25
8501	Valier	Non-recording	52
8597	Virginia City	Non-recording	20
8927	White Sulphur Springs	Non-recording	49
8957	Wilbaux 2E	Non-recording	20
9018	Wilsall	Non-recording	10

[a]U.S. Weather Bureau designation.

[b]Present type of gage--earlier portion of record may be non-recording for recording stations.

[c]Only a portion of the Weather Bureau record, in some cases.



Figure(3): Frequency analysis for Martinsdale, Montana, precipitation station. Hours averaged correspond to duration of rainfall.

curves in Figure (3) do not appear to fit the data very well, confidence curves constructed as described in Chapter II show the goodness of fit to be adequate statistically. These results are rainfall amounts, not rainfall intensities; however, they may be converted to intensity values by procedures outlined below. A summary of the results for all stations is listed in Appendix B, Table (1). Complete results are filed in the Department of Civil Engineering and Engineering Mechanics at Montana State University.

RUNOFF DATA ANALYSIS

Data for the runoff analysis were obtained through the cooperation of the U.S. Geological Survey. As mentioned above, the frequency distribution of rainfall-induced peak flows, the frequency distribution of total peak flows, and the frequency distribution of snow-melt-induced peak flows were required in the evaluation of the factors D and F. In order to evaluate these distributions on a number of watersheds it is necessary to have access to hydrographs of all events on the watersheds, and to be able to separate the discharge due to rainfall, snow melt, and base flow for each hydrograph. In the preliminary stages of this investigation it became evident that currently used hydrograph separation techniques would not be sufficient for this analysis, and that new techniques would have to be developed. As an initial phase of the runoff analysis detailed hydrograph records for several watersheds were obtained for which new separation techniques were developed and tested.

The hydrographs were scanned to determine the pertinent values of

snow-melt-induced flow, rainfall-induced flow, and total peak flow. Frequency curves were obtained from these values by use of the IBM 1620 and SDS Sigma 7 computers. During the course of the initial phase it was found that hydrographs for watersheds larger than 30 square miles area could be generated with a good degree of accuracy from daily discharge records, so that it is unnecessary to resort to original data to obtain hydrographs on watersheds of similar characteristics.

The second phase of the runoff data analysis used daily discharge records on additional watersheds from which hydrographs were generated and separated using the techniques developed in the initial phase. The frequency distributions of rainfall discharge and total peak discharge were obtained for these watersheds using the IBM 1620 and SDS Sigma 7 computers.

The watersheds used in the runoff analysis are listed in Table (4).

Initial technique development phase: Five south-central Montana watersheds were selected for the initial phase of the analysis. The watersheds are the North Fork and the South Fork of the Musselshell River, Sheep Creek, American Fork, and Sweetgrass Creek. Each of these watersheds has been gaged for many years by the U.S. Geological Survey, and continuous records of streamflow are available.

The five watersheds were chosen on the basis of the following criteria:

1) Each watershed has a comparatively long streamflow record.

2) All of the watersheds are located in the same geographical area.

Table (4): List of watersheds and flow measuring stations whose records are used in evaluation of parameters.

Station number[a]	Watershed	Area (sq mi)	Type of gage[b]	Length of record (years)
Watersheds used in first phase of the runoff analysis:				
770	Sheep Cr. (near White Sulphur Springs)	54.4	WSR	11
1155	N. Fk. Musselshell R.	31.4	WSR	25
1185	S. Fk. Musselshell R.	287	WSR	24
1220	American Fork[c]	166	WSR	19
2005	Sweetgrass Creek	63.8	WSR	29
Watersheds used in second phase of the runoff analysis:				
375	Madison R.	420	WSR	20/30[d]
1280	Box Elder Creek	684	WSR	14/15[d]
1545	Peoples Creek	670	WSR	16
1775	Redwater Creek	547	WSR	35
1780	N. Fk. Poplar R.	362	WSR	36
2690	Little Bighorn R.	193	WSR	20/28[d]
3365	Beaver Creek	351	WSR	19/24[d]

[a] U.S. Geological Survey designation.

[b] WSR--water stage recorder.
CSG--crest stage gage.

[c] American Fork data were excluded from analysis after it was learned that important diversions are made above the gaging station.

[d] Discharge due to rainfall determined for portion of record indicated above slash.

3) The watersheds are small in area.

4) Snow melt effects on runoff are minimal.

The USGS loaned their entire set of original water stage records for these five watersheds, together with stage-discharge relations, and records of average daily discharge, to the author. (Obtaining the records was a sizable task, since more than 100 station-years of water stage recorder charts were involved, all of which were shipped from the USGS data storage center in Portland, Oregon.)

After analysis was begun it was learned that the flows from American Fork are regulated by diversions to Lebo Reservoir. No records of these diversions are kept, but they evidently represent a large portion of the runoff from major events on the watershed, so the American Fork watershed was omitted from the analysis.

"Major" runoff events for the remaining four watersheds were noted by examination of the water stage recorder charts. Photostatic copies of the portions of the charts containing these events were made before returning the records to the USGS. The stage discharge relations provided by the USGS were used to plot 228 hydrographs for these events.

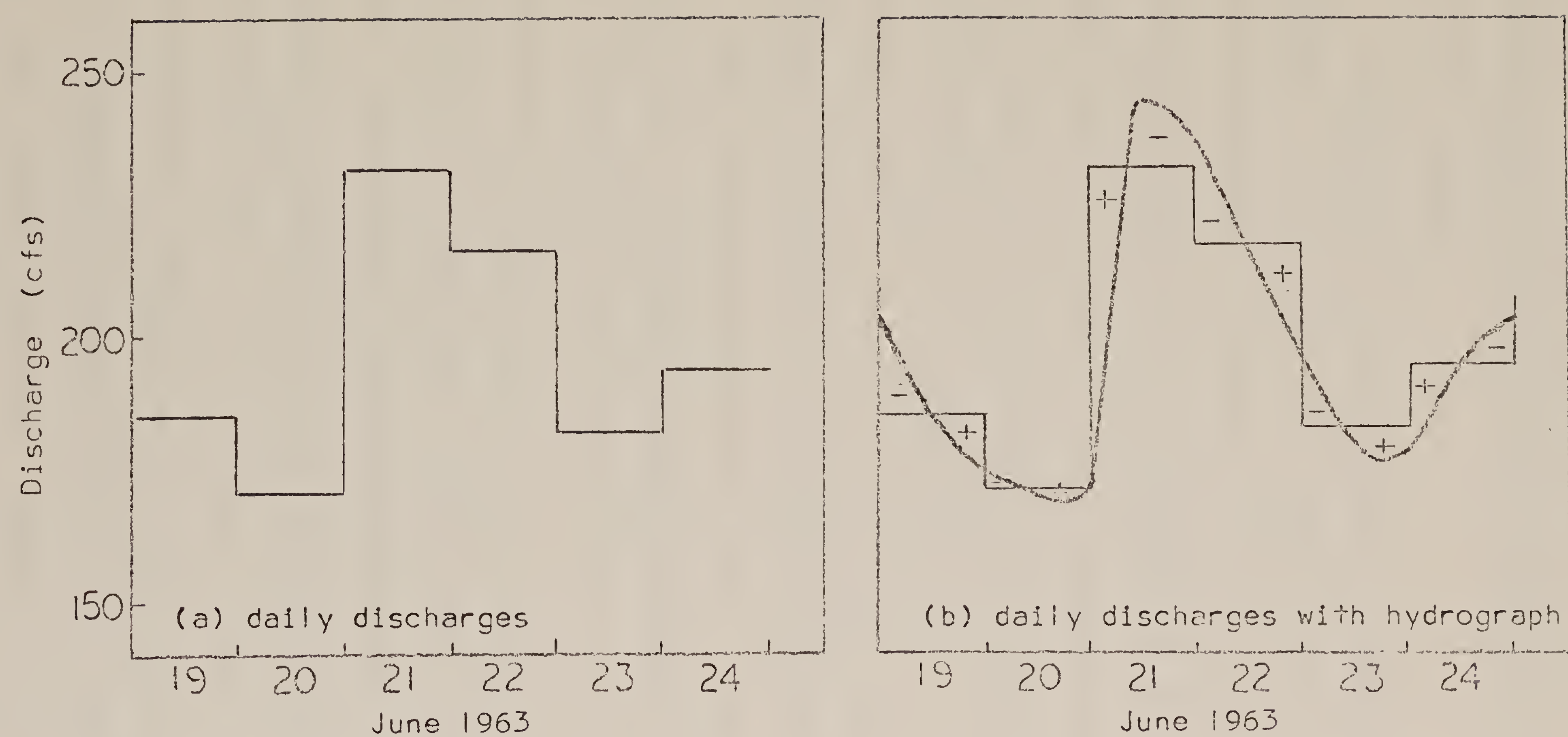
The hydrographs which were produced for the major events on the four watersheds were intended to be used in determining the peak annual discharge due to rainfall and the peak annual discharge due to snow melt during each year of record. Unfortunately, it was found that there were some years during which there were no rainfall events, or during which there were no snow melt events represented among the 228 major-event

hydrographs. Thought was given to borrowing the original recorder charts from the USGS a second time. However, a technique which uses daily discharge values was devised to approximate the true hydrographs. This technique was used in lieu of borrowing the charts again.

The hydrograph generation technique: The hydrograph generation technique may be explained by the following example. Figure (4-a) shows a plot of average daily discharges for one event. Close approximations to the true hydrograph can be made by sketching a line through the plot of Figure (4-a) so that the area between the sketched line and the daily average line adds to zero for each day, if the portion of the area below the daily average line is considered negative, and the portion above the daily average line is considered positive. Figure (4-b) shows the daily average line, together with the sketched line for the event of Figure (4-a). The positive and negative portions of the enclosed area for each day are indicated by plus and minus signs.

Hydrographs composed by this technique generate peak discharges which approximate the true values closely; however, the rising and recession limbs of the hydrographs are less accurately defined. (Comparison was made of generated hydrographs with true hydrographs for a number of events.) Hydrographs approximated by this method were used as needed to supplement the hydrographs of major events in determining peak discharge from rainfall and snow melt events.

Hydrograph separation techniques: The effects of snow melt, rainfall and base flow on the hydrographs obtained above were determined



Figure(4): The hydrograph generation technique applied to the event of June 21, 1963 on the South Fork of the Musselshell River.

using the techniques outlined below. Some of the techniques have not previously been reported in the literature, and were developed especially for this analysis.

A hydrograph for an event may be considered to consist of three portions: 1) base flow, the natural flow of the stream without snow melt or rainfall contributing to the flow (but including interflow); 2) rainfall-induced flow, the portion of the flow occurring as direct surface runoff from rain; and 3) snow-melt-induced flow, the portion of the flow due to melting snow. For a given event any one of the three portions, or any combination of the portions may be present. The techniques used to separate the portions depend on which of the three portions, or combination of portions is present.

When the hydrograph consists only of base flow plus rainfall-induced flow, ordinary hydrograph separation techniques may be applied. The most commonly used technique (and the one used herein) is to extrapolate the base flow curve as a straight line to the point beneath the peak of the hydrograph following the trend of the base flow curve during the period of time immediately before the event began. An example of extrapolation of the base flow curve is illustrated by line (ab) of Figure (5). If the volume of direct runoff were needed, the curve could be extended from the end of line (ab) to an arbitrary point on the hydrograph recession curve by some rule as shown by the dashed line (bc). Various rules are suggested in hydrology texts. Linsley, Kohler, and Paulhus (1958) suggest that . . . "the method of separation

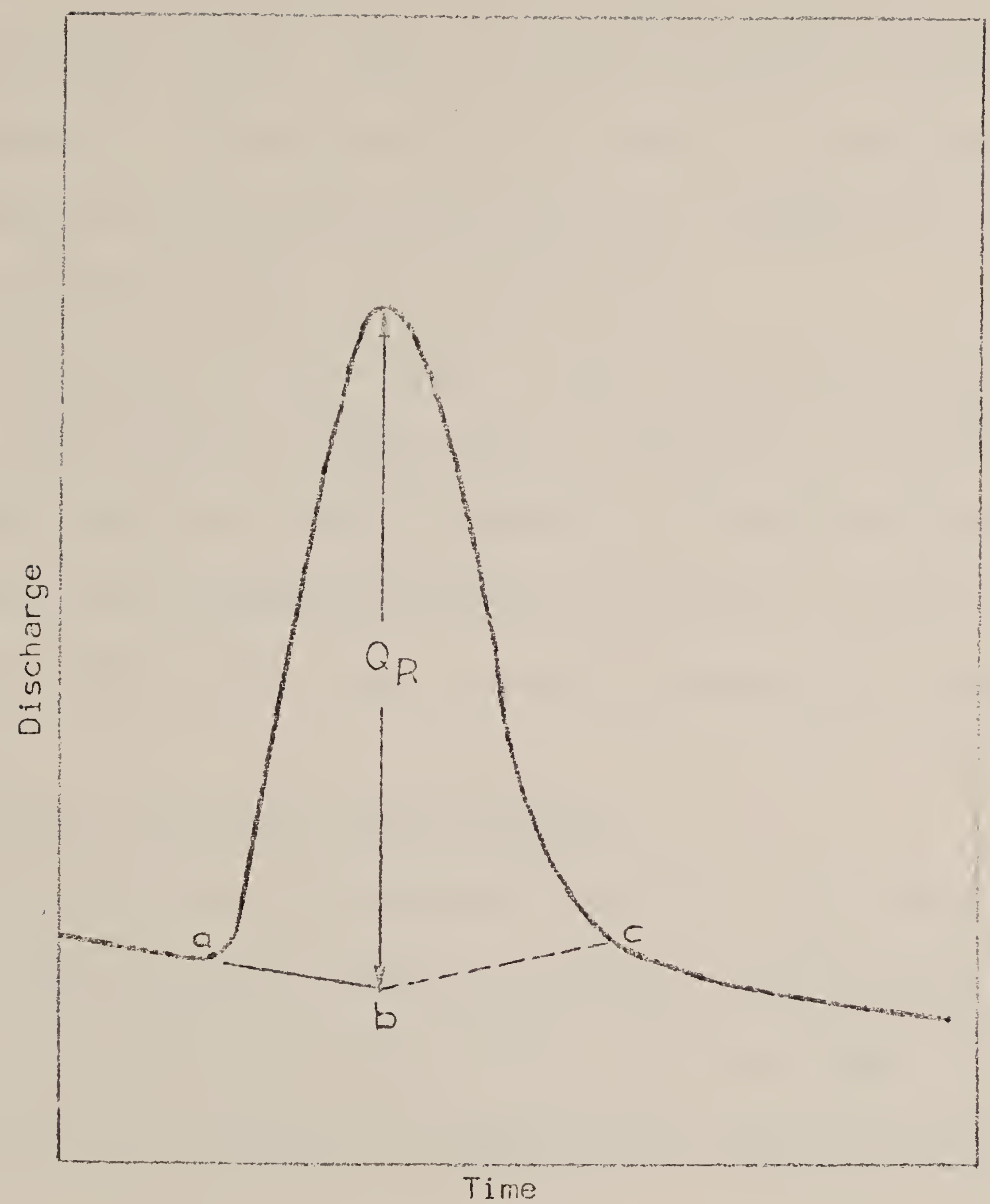


Figure (5): Ordinary technique for hydrograph separation. (ab) represents extension of base flow line prior to event. (bc) represents arbitrary extension to (c).

should be such that the time base of direct runoff remains relatively constant from storm to storm. This is usually provided by terminating the direct runoff at a fixed time after the peak of the hydrograph." Since in this study only the peak discharge due to rainfall, Q_R, was desired the location of point c was not critical.

When the hydrograph consists only of base flow plus snow-melt-induced flow the technique outlined above is ordinarily not applicable, since snow melt events are usually of duration so long that large errors would be present in extrapolating the base flow curve. To give a better estimate of base flow in this case, a special technique was devised by the author.

The technique is based on three concepts:

First, on most streams the base flow (rate) on a given date in one year will not differ greatly from the base flow on the same date on most other years. An average base flow relation constructed using data from many years of record is expected to represent the base flow closely in most cases.

Second, an average base flow relation may be constructed by plotting minimum daily discharge rates occurring during each month of record. The minimum daily discharge rate which occurs during a month will often estimate the base flow (rate) on the date it occurs, since snow melt or rainfall do not usually affect the streamflow during the entire month. The plotted points for the few instances where snow melt or rainfall affect the streamflow during the entire month may be neglected.

Third, the peak seasonal base flow ordinarily occurs later in the year than the peak annual discharge of the stream. This happens because the base flow consists mainly of ground water flow. In an area such as Montana in which most aquifers are composed of unconsolidated material, ground water flow is expected to respond more slowly to precipitation than does surface runoff. This reasoning suggests that the peak of an average base flow curve should generally fall after the median date of the peak annual discharge rate of the stream. The average base flow peak should seldom fall before the median date of the peak annual discharge.

An example of an average base flow relation is shown in Figure (6). Several points on Figure (6) (shown by solid circles) are believed to represent instances where snow melt or rainfall affected the streamflow during the entire month. For example, point (a) on Figure (6) represents a minimum monthly discharge about five times larger than ordinarily occurs in May indicating the presence of factors other than base flow. A trend curve was plotted through the average of the remaining plotted points (shown by open circles) to represent an average base flow relation. The trend curve shown in Figure (6) has been modified slightly so that its peak does not fall before the median date of the mean annual discharge (June 1).

The peak discharge due to snow melt, Q_S, can be determined for events consisting of base flow and snow melt by subtracting the base flow (rate) expected on a given date (as obtained from a relation such

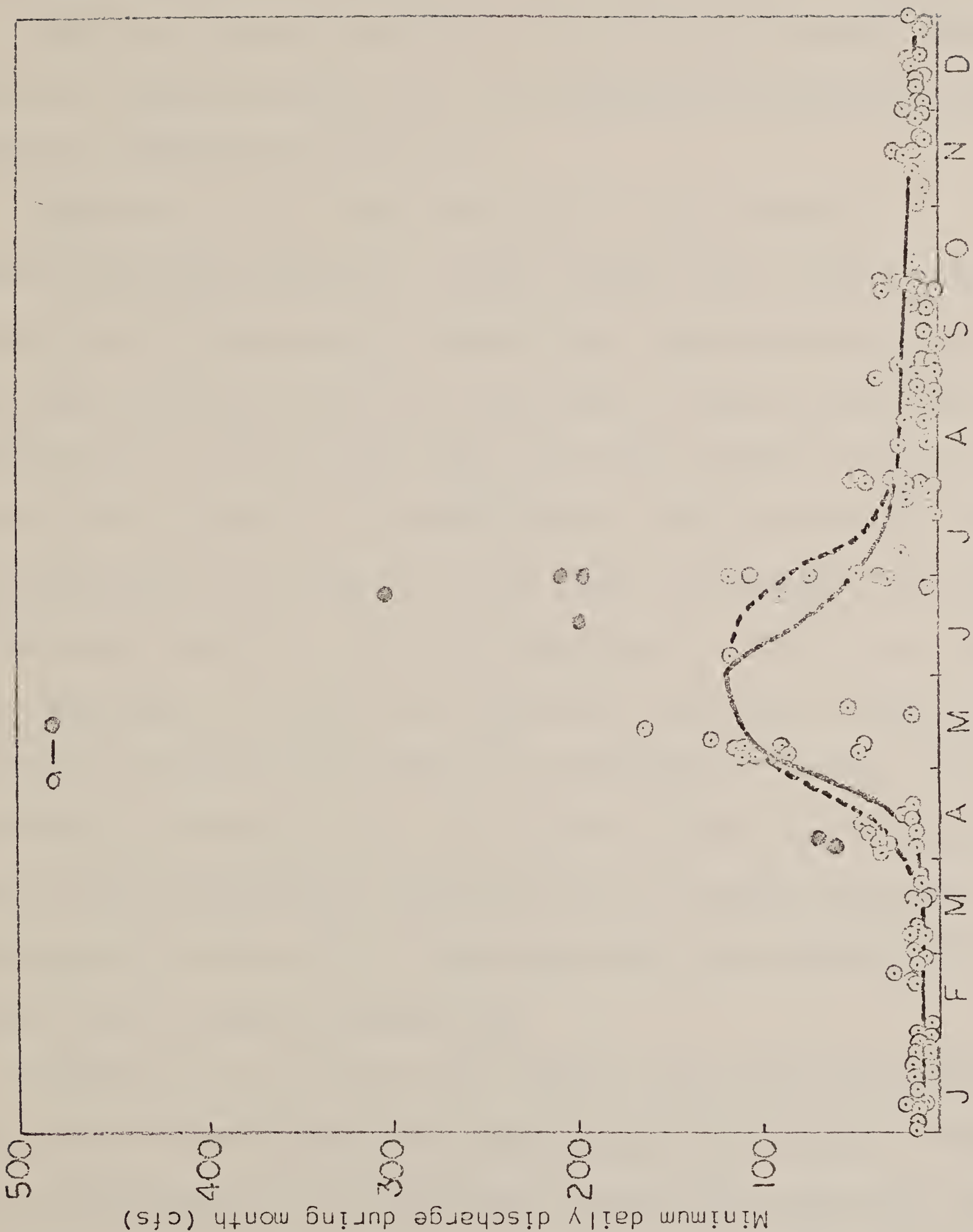


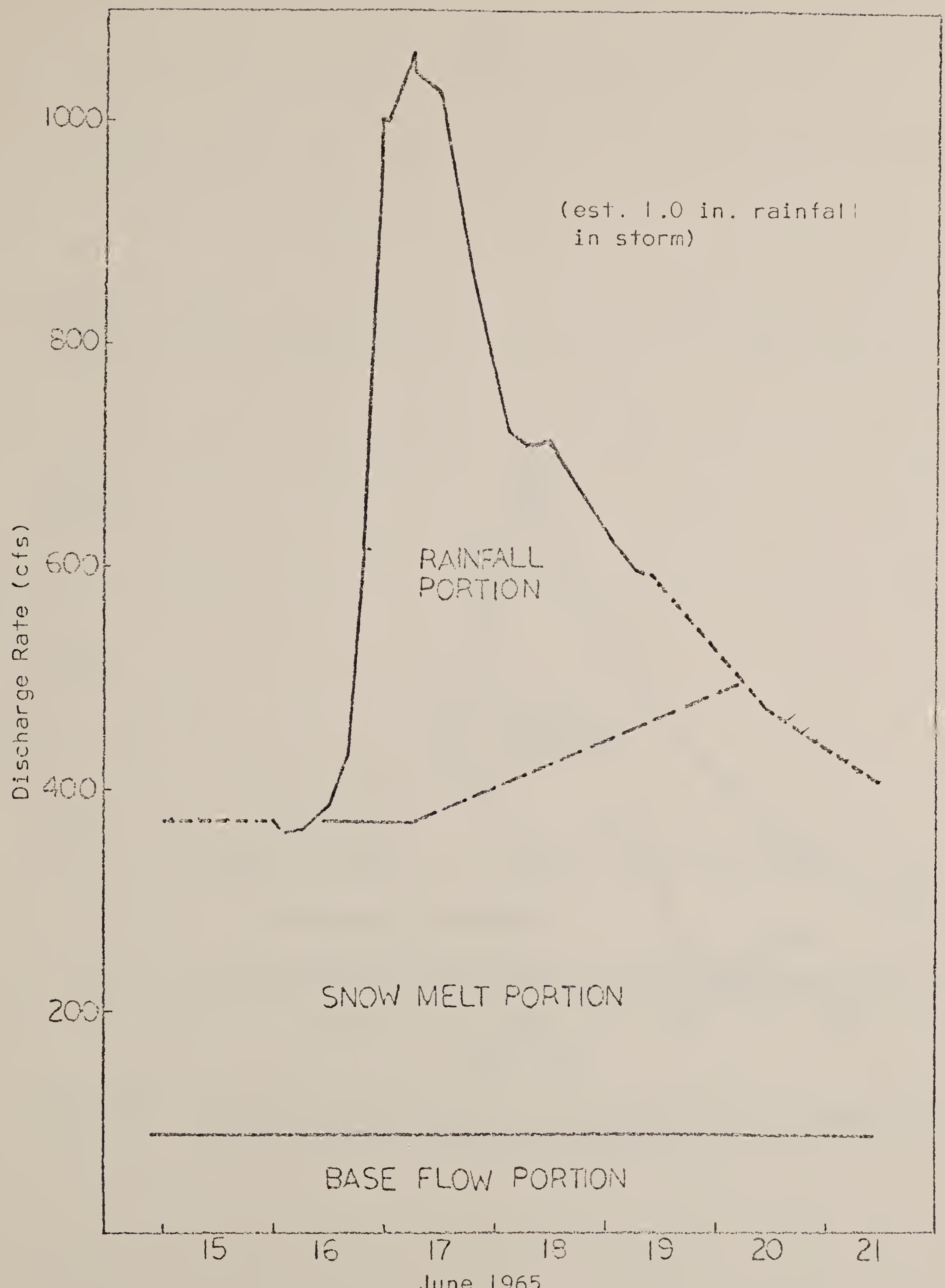
Figure (6): Base flow separation curve for South Fork Musselshell River.

as the solid curve in Figure (6)) from the ordinates of the hydrograph. The maximum difference is Q_S.

When the hydrograph consists of base flow plus snow-melt-induced flow plus rainfall-induced flow, two conditions are possible for which different techniques must be used.

Condition a: If the snow melt runoff on the watershed characteristically appears as a smooth seasonal rise and fall without marked diurnal fluctuation, a rainfall event superimposed upon base flow plus snow-melt-induced flow will appear as shown in Figure (7). In this case the ordinary base flow separation technique outlined above may be used to separate the rainfall portion from the remaining flow, by extrapolating the trend of the base flow plus snow-melt flow curve to the point beneath the peak of the hydrograph. A line is then drawn from that point to a point on the recession curve, such as point (c) on Figure (7), in accordance with the suggestions of Linsley, Kohler, and Paulhus, previously cited. The base-flow portion may be separated from the snow-melt-induced portion by use of a seasonal base flow relation such as Figure (6). Separation of the three portions for a typical event is shown in Figure (7).

Condition b: If the snow melt runoff on the watershed has diurnal fluctuation or has several peaks during the snow melt season a rainfall event superimposed on a base-flow plus snow-melt flow event will appear as shown in Figure (8). In this case the trend of the base flow plus snow-melt flow curve is not easy to extrapolate. Separation of the rainfall portion from the remaining flow in this case requires the



Figure(7):Base flow, snow melt, and rainfall separation for the event of June 17, 1965 on the South Fork of the Musselshell River.

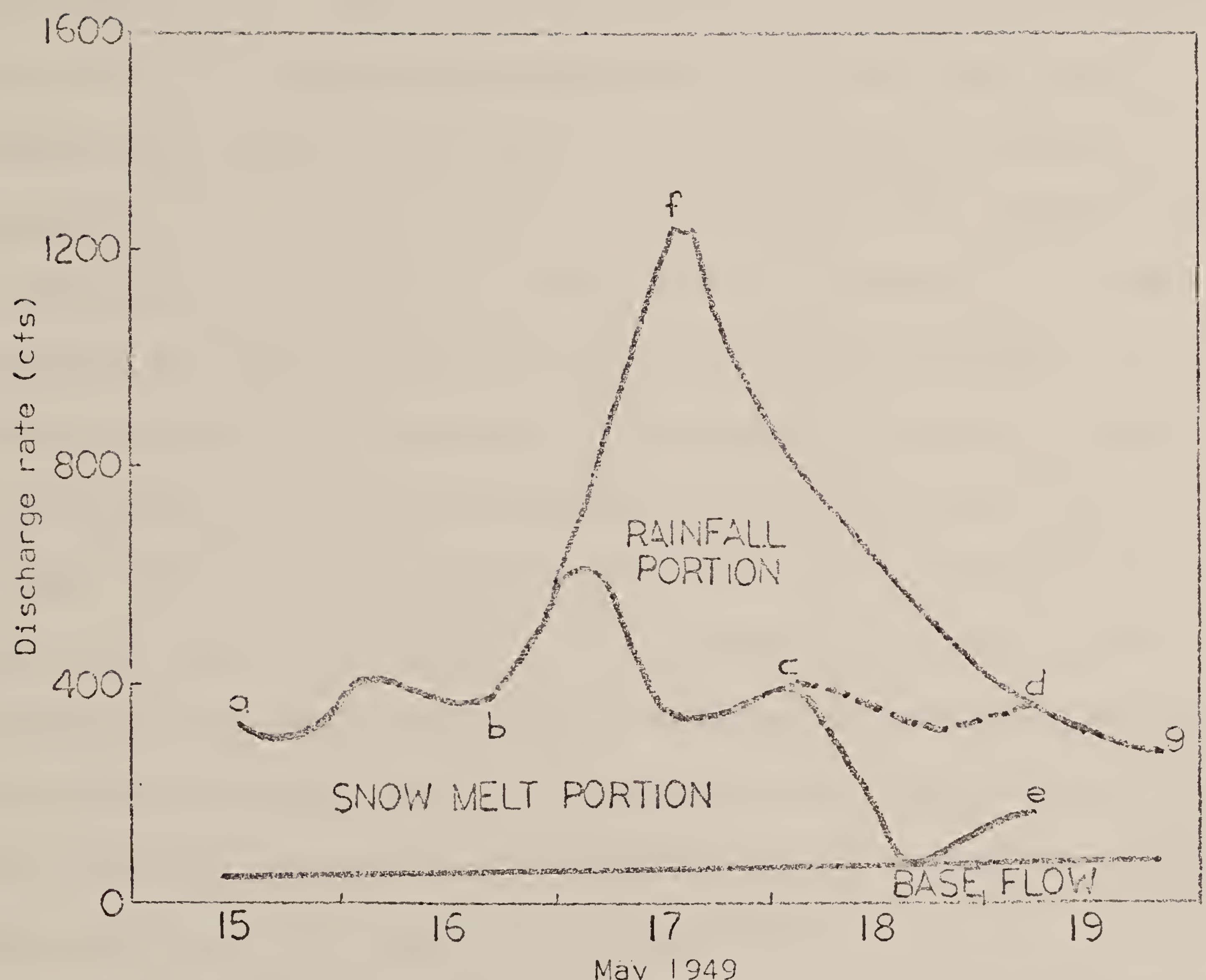


Figure (8): Base flow and snow-melt flow separation for event of May 17, 1949 on Sweetgrass Creek.

introduction of a new concept as described below.

The base-flow plus snow-melt flow portion of the hydrograph during a rainfall induced event might be expected to be similar in shape to hydrographs for the watershed which consist only of base flow and snow-melt-induced flow. This concept may be used as an aid in extrapolating the trend of the base-flow plus snow-melt flow curve, and hence in obtaining the desired separation of the rainfall induced portion. A convenient plot for this purpose is shown in Figure (9). Figure (9) was constructed by plotting the discharge-time relationship for a number of hydrographs consisting only of base flow and snow-melt-induced flow, using the peak point of the hydrograph as the origin of the graph coordinates. It was found that on a given watershed, the base-flow plus snow-melt-induced flow hydrographs tended to fall into one of several general types. Hence, in Figure (9) only three curves (called type curves in the following discussion) were needed to represent all base-flow plus snow-melt-induced flow hydrographs on Sweetgrass Creek. No more than three type curves were found to be necessary for any one watershed. One type curve was found to be adequate for Sheep Creek.

To extrapolate the base-plus-snow-melt flow curve using type curves, the type curve which best fits the trend of the base-plus-snow-melt flow curve prior to the rainfall event is selected. When possible the type curve should fit the trend of the base-plus-snow-melt flow curve after the rainfall event as well. On many watersheds the peaks of snow melt hydrographs usually occur at about the same hour of the

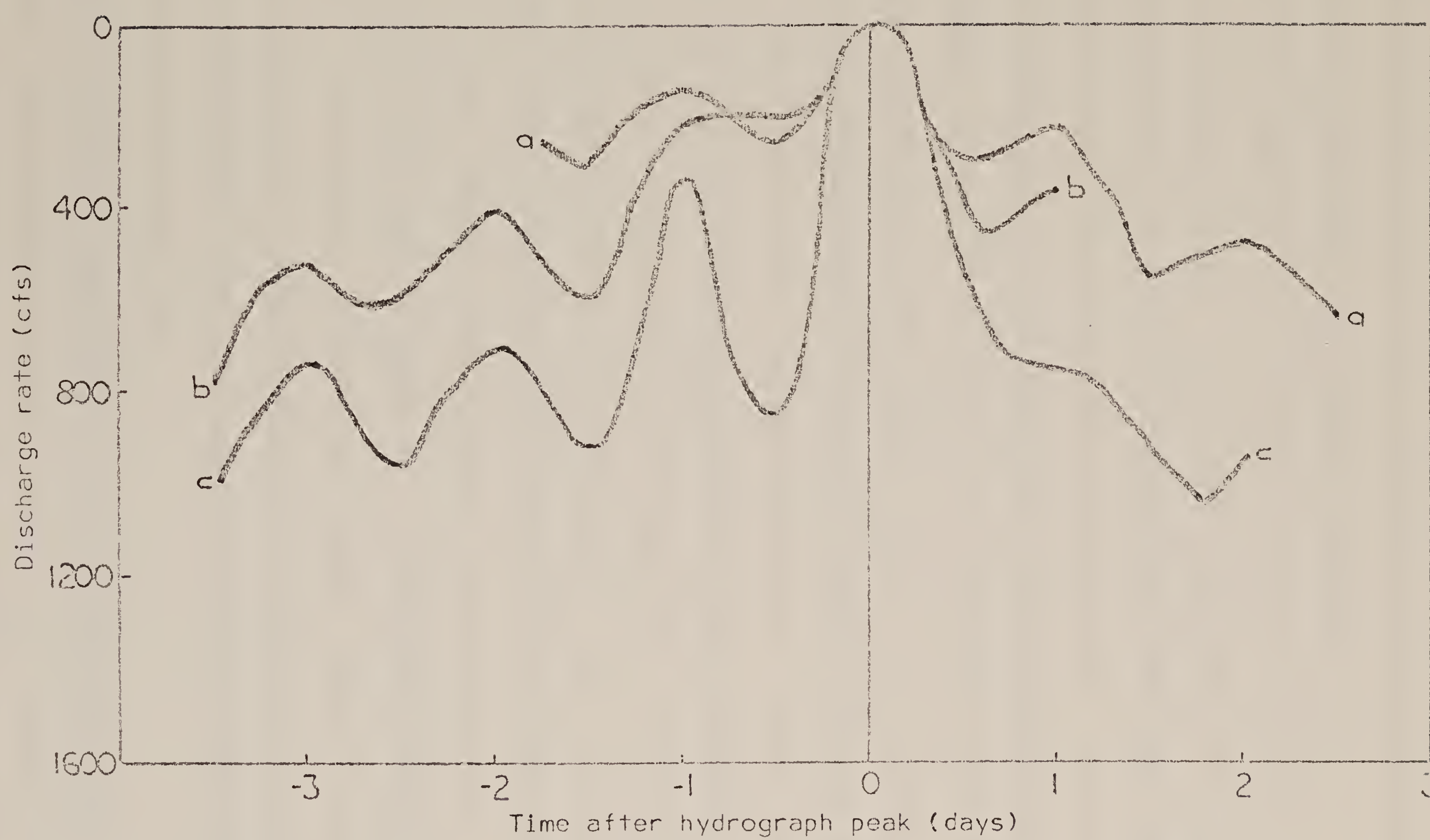


Figure (9): Superimposed plots of time-discharge relations (type curves) showing typical shapes of snow melt hydrographs on Sweetgrass Creek.

day. This fact may be used to help locate the various type curves correctly on the time axis, when making a selection.

Figure (8) illustrates the selection and use of a type curve for an event on Sweetgrass Creek. The base-plus-snow-melt flow curve preceding the event, line (ab), and following the event, line (dg) are to be fitted by one of the type curves from Figure (9). Since snow melt peaks usually occur at about 3:00 AM at the Sweetgrass Creek gaging station, the peak of the type curve should fall at 3:00 AM on one of the days. Examination of the general trend of the base-plus-snow-melt flow curve should indicate whether the rainfall occurs on the rising or falling limb of the snow melt hydrograph. In this case, since the curve is generally flat the snow melt peak evidently occurs during the rainfall event. The snow melt peak, then, could occur either at 3:00 AM, May 17, or at 3:00 AM, May 18. Upon trying the various type curves, it is found that type curve (a) on Figure (9) fitted so its peak falls at 3:00 AM, May 17, gives the best fit. This fit is shown by line (bce) on Figure (8). In this case the type curve does not fit the condition following the rainfall event. The dashed line (cd) represents an arbitrary adjustment to make the curve fit the condition following the event. The adjustment does not affect the value of Q_R determined for this event.

Analysis of initial phase data: Using the hydrograph separation results peak annual discharges due to rainfall, Q_R, (excluding base flow) and peak annual discharges due to snow melt, Q_S, (excluding base

flow) were determined for each year of record. The peak annual discharge regardless of cause, Q_T, (with base flow included) was obtained for each year of record from USGS publications (U.S. Geological Survey, 1960, etc., and 1966, etc.). Extreme value frequency curves were fitted to the data by use of the IBM 1620 and SDS Sigma 7 computers (programs listed in Appendix A). A graph showing the frequency curves for one watershed is shown in Figure (10). (The extreme value fit does not account for the fact that the peak annual discharge cannot be negative, and, thus, it may cross the zero axis. Practically, the fitted line must be considered to end at a discharge of zero, e.g. the Q_R curve on Figure (10).) Parameters describing the curves for all of the watersheds are listed in Table (2), Appendix B. Complete results of the frequency analyses are on file in the Department of Civil Engineering and Engineering Mechanics at Montana State University.

Second phase analysis and results: For the second phase of the runoff analysis six additional watersheds located in Montana east of the continental divide were selected. The watersheds are the Madison River, the Little Bighorn River, Redwater Creek, the North Fork of the Poplar River, Peoples Creek, and Beaver Creek near Wibaux. These watersheds were selected on the basis of the following criteria:

1) Each watershed has a comparatively long streamflow record.

2) The watersheds represent various geographical areas of Montana east of the continental divide.

3) The watersheds are essentially unaffected by diversions.

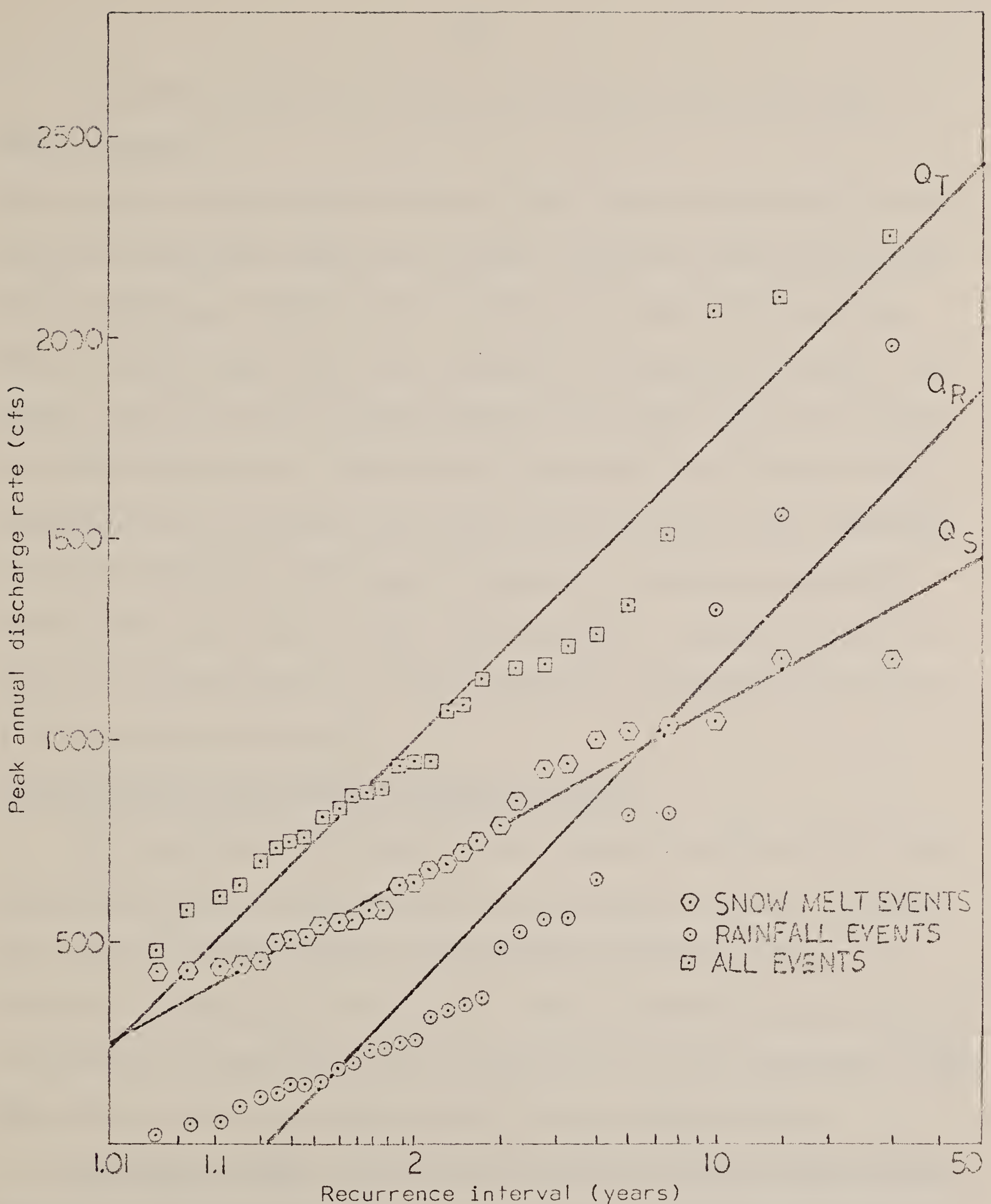


Figure (10): Recurrence relation for rainfall events, snow melt events, and all events on Sweetgrass Creek.

4) The watersheds are as small as possible to meet the first three criteria.

Hydrographs on these watersheds were formed from daily discharge records using the hydrograph generation technique described above and separated by the techniques outlined above to obtain the peak annual discharge due to rainfall, Q_R, (base flow excluded) for each year of record. As before, the peak annual discharges due to all causes, Q_T, were obtained from USGS publications. Extreme-value frequency curves for peak annual discharges due to rainfall were fitted to the data for these watersheds using the IBM 1620 and SDS Sigma 7 computers. Parameters describing these curves are listed in Table (2), Appendix B. Complete results are on file in the Department of Civil Engineering and Engineering Mechanics at Montana State University.

DETERMINATION OF THE RAINFALL INTENSITY RATIO, R

The results from the rainfall data analysis together with data from Hershfield (1961) and U.S. Weather Bureau Technical Paper No. 28 (1956) were used to determine values of the rainfall intensity ratio, R, for durations ranging from 1 to 24 hours. All the available values were used to construct an isoplethal map for Montana east of the continental divide. The methods used in the determination of R are outlined below.

Conversion of data: Recall from a previous section in this chapter that the analysis of rainfall data consisted of determining, for eight recording rain-gage stations, the maximum annual amounts of rain that fell during fixed observational intervals of 1 hour, 2 hours, 3 hours,

4 hours, 6 hours, 8 hours, 12 hours, and 24 hours; for 31 non-recording rain-gage stations, the maximum amounts of rain that fell during fixed observational intervals of 24 hours. "Fixed observational intervals" here refer to the fact that consecutive blocks of time were used in the determination. For example, to determine the maximum annual one hour amount of rainfall, the recording rain gage data were examined for one hour intervals such as 12:00 M to 1:00 AM, 1:00 AM to 2:00 AM, 2:00 AM to 3:00 AM, etc.; to determine the maximum annual two hour amount of rainfall, the data were examined for two hour intervals such as 12:00 M to 2:00 AM, 2:00 AM to 4:00 AM, 4:00 AM to 6:00 AM, etc.; to determine the maximum three hour amount of rainfall the data were examined for three hour intervals such as 12:00 M to 3:00 AM, 3:00 AM to 6:00 AM, 6:00 AM to 9:00 AM, etc. The extreme value (Gumbel) recurrence analysis was applied to the results to obtain the mean annual rain and the 50-year rain for the observational intervals indicated. These values are tabulated in Table (1), Appendix B.

The defining equation for the rainfall intensity ratio is

$$R = I^{i} / \bar{I} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (28)$$

where I^{i} is the rainfall intensity (for a rain of given duration) with the same recurrence interval as the i-year flood (in this analysis i is taken as 50 years), and $\bar{I}$ is the mean annual peak rainfall intensity (for a rain of the same duration). Equation (28) requires values of rainfall intensity (rather than rainfall amounts) based on "true"

intervals beginning at any point in time. For example, the maximum three hours of rain which fell at Martinsdale in 1956 may have started at 1:33 AM and ended at 4:33 AM (rather than starting at 12:00 M and ending at 3:00 AM, or starting at 3:00 AM and ending at 6:00 AM).

Reflection on the above comments indicates that the average maximum observational interval rainfalls determined in this study are in most cases less than the corresponding average maximum true interval rainfalls, and, therefore, that a conversion is needed.

To convert fixed-interval rainfall amounts to true interval rainfall amounts a statistical relation developed by Weiss (1964) expressly for this purpose was used. To develop his relation, Weiss assumes that the rainfall intensity is constant during the true interval; hence, the amount of precipitation during the interval is proportional to the length in probability considerations. He then considers a line of unit length divided into subintervals by points chosen at random. Since the true interval may begin (at random) at any point within the observational interval, the probability that none of the subintervals is less than x in length can be used to develop the desired conversion factor. Weiss shows that the reciprocal of the expected value of x over the unit length of line (numerically equal to 1.143) is the correct conversion factor. Thus,

$$\frac{\text{Average maximum true interval rainfall}}{\text{Average maximum observational interval rainfall}} = 1.143 \qquad (29)$$

This conversion factor is applicable for any given length of interval.

An empirical relation of essentially the same numerical value has been developed independently by Hershfield (1961) which he applies to widely varied geographical areas, including Montana.

To test the validity of Weiss's relation for Montana, values of his coefficient were estimated empirically using rainfall data from 5387, Martinsdale. The test was accomplished with the aid of a modification of the computer program listed in Appendix A[1]. The modified program gives rainfall amounts based on overlapping intervals (e.g. 12:00 M to 3:00 AM, 1:00 AM to 4:00 AM, etc.) rather than consecutive intervals as does the original program, (e.g., 12:00 M to 3:00 AM, 3:00 AM to 6:00 AM, etc.). For the one-clock hour intervals (e.g. 12:00 to 1:00 AM, 1:00 AM to 2:00 AM, etc.) the two programs give the same annual maximum one hour rainfall; but for each increasingly longer interval the results from the modified program should more closely approximate the true interval rainfall, and for the 24-hour interval should give a quite accurate estimate of the true interval rainfall. Weiss's relation was tested by dividing the rainfall amounts obtained from the modified program by the corresponding amounts obtained from the original program at the mean and 50-year levels. The ratios resulting are estimates of Weiss's relation. These estimates are shown in Table (5). For verification of Weiss's relation, the values are expected to increase from 1.00 for the one-hour interval to approximately 1.143 for the 24-hour interval. Since the values in Table (5) follow the

[1] The program was modified by T. T. Williams and T. L. Hanson.

Table (5): Empirical estimates of Weiss's relation for Martinsdale, Montana.

Length of interval (hours)	Estimate for mean annual rain	Estimate for 50-year rain
1	1.00	1.00
2	1.07	1.04
3	1.00	1.03
6	1.00	0.90
8	1.13	1.02
12	1.18	1.06
24	1.21	1.13

expected trend reasonably well, Weiss's relation is considered to be valid for use herein.

Maximum true interval rainfall amounts were obtained from the values given in Table (1), Appendix B, by use of Weiss's relation. For example, the 50-year rainfall amount for Barber during 24 hours is given in Appendix B as 1.92 in. and the mean-annual 24-hour rain is given as 0.99 in. These values correspond to maximum true-interval rainfall amounts of 1.92 x 1.143 = 2.19 in. for the 50-year rain, and 0.99 x 1.143 = 1.131 for the mean annual rain.

To obtain average maximum true-interval rainfall intensities, the maximum true-interval rainfall amounts were divided by the observational interval. For example, the true-interval 50-year rainfall intensity with duration 24 hours for Barber is 2.19/24 = 0.0913 in./hr, and the corresponding mean annual 24-hour intensity is 1.131/24 = 0.0472 in./hr.

Computation of R: The 50-year maximum true interval rainfall intensities divided by the mean-annual true interval rainfall intensities, according to Equation (28), give values of R. Thus, for Barber, 0.0913/0.0472 = 1.94 is the value of R for duration 24 hours.

The proper duration over which the true interval intensity should be averaged to compute the correct values of R must be considered. The duration of a given rainstorm has a significant effect on the peak discharge produced. A certain period of time (called travel time) is required for flow induced by the rainstorm to travel from the farther

end of the watershed to the mouth of the watershed. If the duration of the rain is equal to or greater than the travel time of the watershed, flow from all portions of the watershed will have reached the mouth of the watershed before the rain ceases, and the peak flow can be expected to be larger than that produced by a storm of the same intensity but of shorter duration. The length of time (observational interval) over which the true interval rainfall intensity should be averaged to correspond to discharge should, then, be an amount close to the value of the travel time for a given watershed.

Examination of Figure (11) shows that if the averaging time for true interval rainfall intensity, i.e., the travel time of the watershed, is equal to or greater than about twelve hours the value of R is essentially constant with observational interval. Daily precipitation values can be used, then, to determine values of R corresponding to Q_R for any watershed with travel time greater than about 12 hours.

Values of R determined for the precipitation stations studied herein were supplemented by values obtained from data given in Hershfield (1961) and U.S. Weather Bureau Technical Paper No. 28 (U.S. Weather Bureau, 1956). Hershfield gives isoplethal maps of rainfall amounts of given duration for 1, 2, 5, 10, 25, and 100 year recurrence intervals while T.P. 28 gives isoplethal maps of the ratio of 100-year to 2-year rainfall intensities for durations of 6 hours and 24 hours. The mean annual (2.33-year) rainfall amounts for 24 hour durations were found for selected

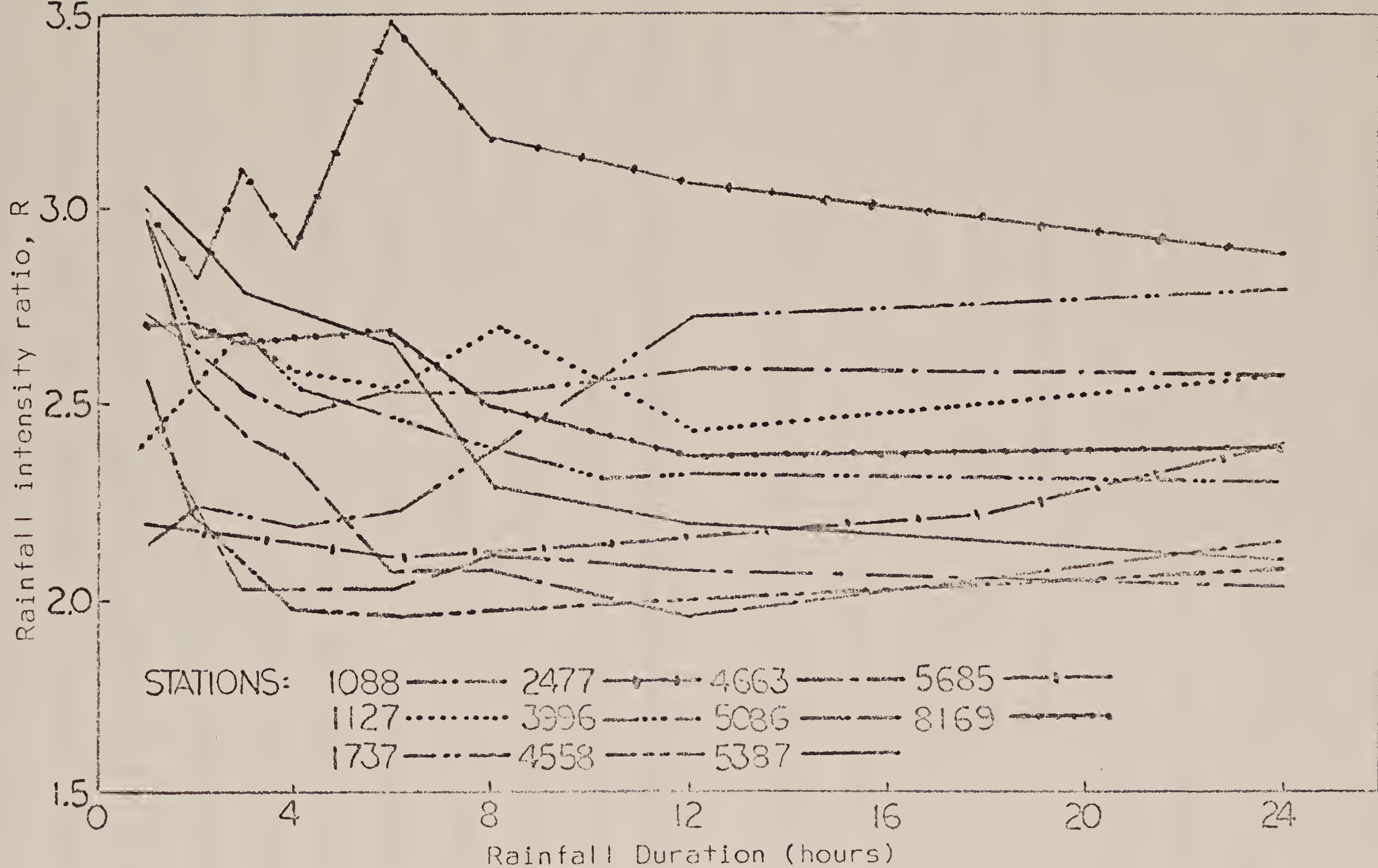


Figure (11): Variation of R with observational interval (rainfall duration).

locations on the Hershfield maps by interpolation between the 2-year and 5-year values, and values of R were obtained by dividing 50-year amounts by corresponding mean-annual amounts. Values of R for 24-hour durations were obtained for selected locations on the T.P. 28 maps by noting that $I^{50}/I^{2.33}$ values (R) are in each case proportional to the corresponding I^{100}/I^{2} values.

All available values of R for the 24-hour duration were used to prepare Figure (12). Numbers on the figure are the values computed for the precipitation stations studied herein. Values for some short-record stations, which may not be representative, are omitted.

DETERMINATION OF THE RAINFALL-DISCHARGE RECURRENCE FACTOR, D

Values of the rainfall-discharge recurrence factor, D, were obtained for the watersheds analyzed in the runoff data analysis by use of Equation (23). In Equation (23), C_{vQ} was obtained as the coefficient of variation of the peak annual rainfall-induced discharges (standard deviation divided by the mean) for each watershed. Values of C_{vQ} are listed in Table (2), Appendix B. Values of σ_{nQ}, $\bar{y}_{nQ}$, σ_{nI}, and $\bar{y}_{nI}$ were obtained from Linsley, Kohler, and Paulhus (1958). σ_{nQ} and $\bar{y}_{nQ}$ were taken as 0.95 and 0.51 respectively, (for record length 10 years) since short streamflow records are used. σ_{nI} and $\bar{y}_{nI}$ were taken as 1.28 and 0.57 (for an infinite record) since long rainfall records are used. An average value of C_{vI} for Montana east of the continental divide (43.8%)

Figure (12): Isoplethals of the rainfall intensity ratio, R, for Montana east of the continental divide.

was used, as shown in Table (1), Appendix B. The variation of C_{vI} within eastern Montana is small so that the error made by using the average value is insignificant. (Fifty-seven percent of the C_{vI} values fall within 10 percentage points of the mean, with most of the values further from the mean resulting from short records.) The values of D computed from Equation (23) were used to form an isoplethal map applicable to Montana east of the continental divide. The map is shown in Figure (13). Dashed portions of the isoplethals are partially estimated and should be used with caution.

DETERMINATION OF THE RAIN-SNOW-BASE FLOW INTERACTION RATIO, F

Equation (24) was used to determine values of the rain-snow-base flow interaction ratio for each of the watersheds analyzed in the runoff data analysis (values listed in Table (2), Appendix B). Values were found to vary from 0.93 to 1.84 with an average value of 1.45.

Since the F values have a comparatively wide range of magnitudes, attempts were made to correlate F with factors which could possibly reflect the interaction between the frequency curves of base-flow, snow-melt-induced flow, and rainfall-induced flow. It was found that F is not correlated to the geographical location of the watershed, to the area of the watershed, nor to the elevation of the gaging station. Neither is it correlated to the median date of the mean annual flood, the length of the frost free season, nor the mean date of the last freeze on the watershed. No correlation exists between F and the percentage of mean annual floods having rainfall assocaited with them.

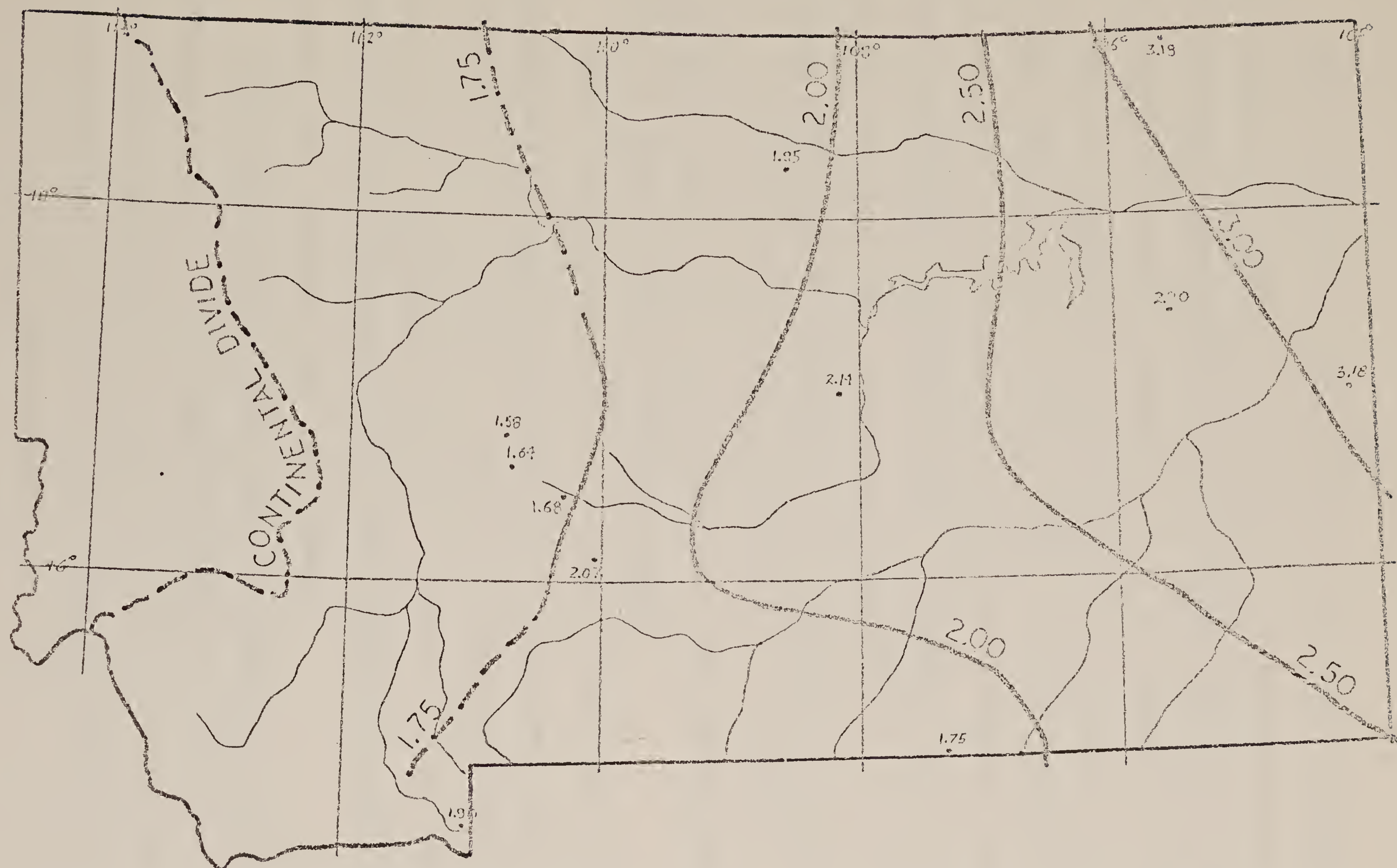


Figure (13): Isoplethals of the rainfall-discharge recurrence factor, D, for Montana east of the continental divide.

Since F could not be related simply to any of these variables, it appears that the interaction is too complex to be expressed simply. For this reason it was decided to use the average value of F as being representative for Montana watersheds east of the continental divide.

Use of an average value of F is similar to assumptions used by Dalrymple (1960) and Boner and Omang (1967). To illustrate this, note that on Figure (12) and Figure (13) R and D may be considered constant within a homogeneous geographical region. By Equation (25), if F is considered constant within a homogeneous region $Q_T^i/\bar{Q}_T$ will be constant. Assuming $Q_T^i/\bar{Q}_T$ constant is analogous to the assumption of Dalrymple that a composite-frequency curve is applicable to a homogeneous geographical region, and to the assumption of Boner and Omang that Q_T^{25}/Q_T^{10} is constant within a homogeneous region.

Since F reflects the relative independence between the frequency distributions of rainfall-induced discharge and snow-melt-induced discharge, it is of interest to examine the relative independence of these curves. The frequency curves may be compared for the four watersheds of the first phase of the runoff analysis, for which the snow-melt flow frequency curves were obtained.

If the value of the snow-melt induced discharge, Q_S, were completely independent from the rainfall-induced discharge, Q_R, the larger of the two values would determine the total discharge, Q_T, at any given recurrence interval. If Q_S and Q_R were completely dependent, Q_T would equal Q_S plus Q_R (again neglecting base flow) at any given

recurrence interval.

Comparison of Q_S and Q_R for the four watersheds (see Table (2), Appendix B) shows that the values are essentially dependent at the mean-annual level, while they are more nearly independent at the 50-year level. For example, on Sweetgrass Creek $\bar{Q}_S + \bar{Q}_R = 1178$ cfs, while $\bar{Q}_T = 1069$ cfs, with the difference the same order of magnitude as base flow, indicating complete dependence. $Q_S^{50} + Q_R^{50} = 3293$ cfs, while $Q_T^{50} = 2406$ cfs, with the difference of greater order of magnitude than the base flow, indicating some independence.

The increase of independence of Q_S and Q_R with increasing recurrence interval suggests that F may be a function of recurrence interval and that the value determined might be invalid if based on a very long recurrence interval (e.g., i = 200 years).

CHAPTER V

TEST OF THE RAINFALL-DISCHARGE INDUCTION METHOD

Many theoretical frequency distributions have been proposed by various investigators to fit peak discharge data. To establish whether one of these theoretical distributions is more nearly correct than the others would require a large sample from the population of peak flow values consisting, perhaps, of several thousand years of record. Until such long records become available the correctness of any theoretical distribution will be subject to conjecture. The extreme value distribution and the log-normal distribution are used widely by hydrologists to estimate the distribution of peak discharges that have been shown to agree in many cases with other commonly used theoretical distributions. For these reasons peak discharge values estimated by the rainfall-discharge induction method (developed in this study) will be compared with values estimated from the extreme value distribution and the log-normal distribution. It should not be implied, however, that these distributions are more nearly correct than other distributions.

To check the validity of Equation (16) only values of $\bar{Q}_R$ and Q_R^{50} estimated by the extreme value recurrence analysis for the twelve watersheds used in the runoff data analysis were used (see Table (4)).

Figure (14) compares Q_R^{50} values estimated from the best-fit extreme value (Gumbel) recurrence curves for the twelve watersheds, with values obtained from Equation (16) using D values obtained from Figure (13). The numbers used to identify the watersheds are those assigned by the

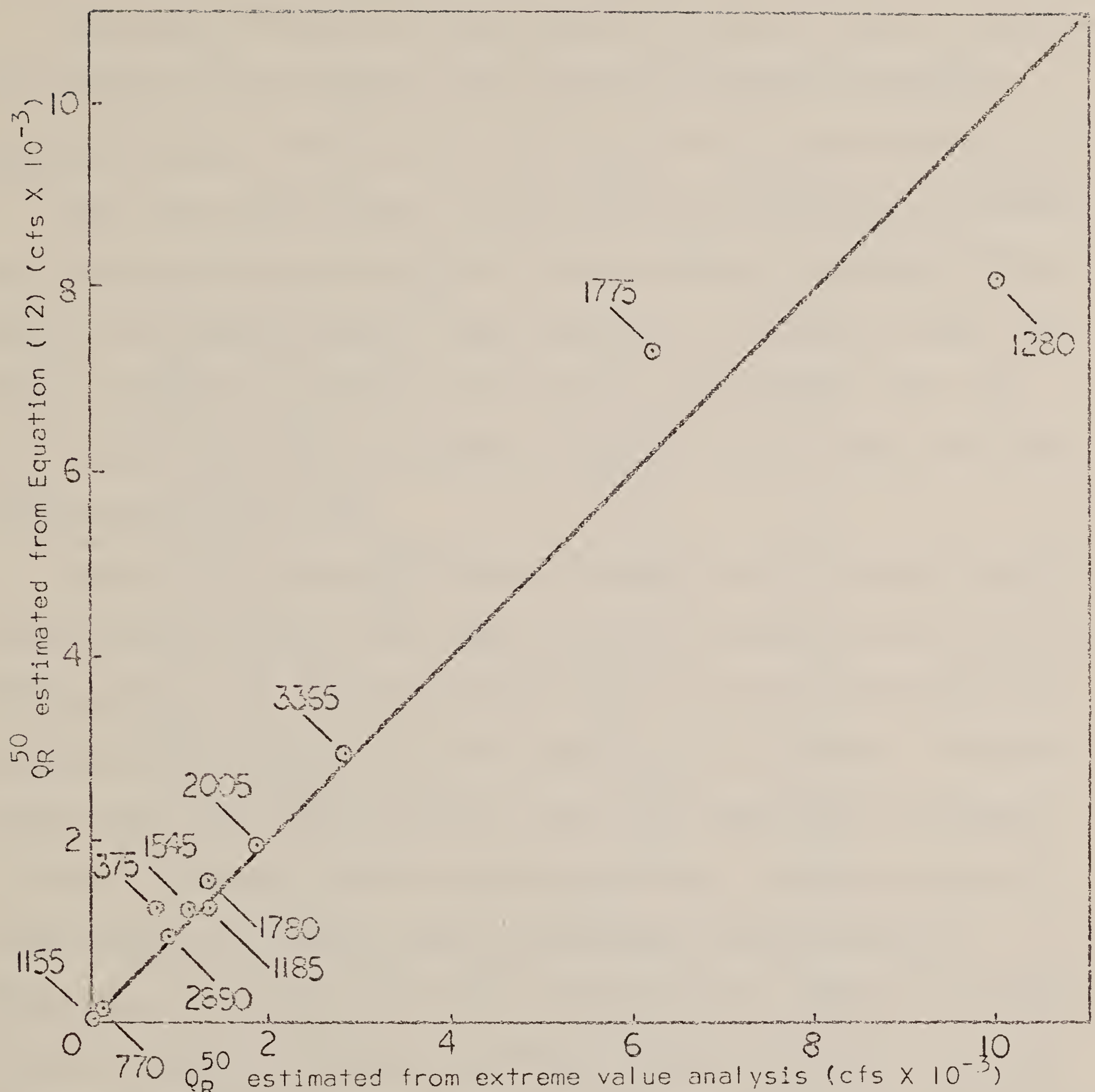


Figure (14): Comparison of estimates of Q_R^{50} from Equation (16) with estimates from extreme value recurrence analysis.

U.S. Geological Survey, as indicated in Table (4).

The points of Figure (14) generally fall along the line of equality. This implies that the extreme value recurrence analysis of rainfall induced discharges estimates the same set of values as does Equation (16).

Peak flow data from Boner (1967) for 46 crest-stage-gaged watersheds were used as a further test of the rainfall-discharge induction method. The area and record length of these watersheds are listed in Table (6). Values of Q_T^{50} were computed using values of D from Figure (13), and $F = 1.45$ for all of the watersheds, in Equation (27). Values of R were obtained from Figure (12).

Figure (15) compares Q_T^{50} values estimated from the best-fit extreme value recurrence curves with values estimated from Equation (27) for the 46 watersheds. For several watersheds the extreme value and log-normal estimates differ significantly from Equation (27) estimates. In one of these cases, 1209, Antelope Creek, much of the difference is believed to be due to large, unusual events which dominate the other events of record making the extreme value and log-normal analyses invalid. Examination of the data from 1771.5, Redwater Creek, indicates that the computed estimate of $\bar{Q}_T$ may be too high, thereby causing the rainfall-discharge induction method to produce a Q_T^{50} value that is too large. The method gives estimates close to the extreme value and log-normal estimates, however, on tributaries of both Antelope Creek and Redwater Creek. On 1395, Big Sandy Creek, 1765, Wolf Creek, and 1830, Big Muddy Creek, several of the largest events of record were caused by snow melt. It is

Table (6): List of watersheds and flow measuring stations whose records are used in checking parameters.

Station number[a]	Watershed	Area (sq mi)	Type of gage[b]	Length of record (years)
135	Sheep Cr. (near Dell)	280	CSG	15
155	Grasshopper Creek	348	CSG	30
465	E. Gallatin River	49	CSG	10
470	Bear Canyon Creek	17	CSG	10
760	Newland Creek	6.74	CSG	15
893	Sun River trib.	21.1	CSG	11
1003	Lone Man Coulee	14.1	CSG	7
1022	Marias R. trib.	1.62	CSG	11
1023	Marias R. trib. No. 2	0.25	CSG	11
1206	Antelope Cr. trib.	0.47	CSG	11
1207	Antelope Cr. trib.	1.92	CSG	11
1208	Bacon Creek	21.2	CSG	11
1209	Antelope Creek	88.7	CSG	14
1289	Box Elder Cr. trib.	16.2	CSG	12
1295	McDonald Creek	421	CSG	28
1297	Gorman Coulee	2.32	CSG	11
1298	Gorman Coulee trib.	0.81	CSG	12
1306	Cat Creek	36.5	CSG	9
1308	Second Creek trib.	1.90	CSG	10
1395	Big Sandy Creek	1805	CSG	21
1552	Alkali Creek	162	CSG	9
1553	Disjardin Coulee	3.42	CSG	11
1554	S. Fk. Taylor Cr.	5.08	CSG	11
1765	Wolf Creek	251	CSG	22
1770.5	E. Fk. Duck Cr.	12.4	CSG	12
1771	Duck Creek	54.0	CSG	10
1771.5	Redwater Creek	216	CSG	10

[a]U.S. Geological Survey designation.

[b]WSR--water stage recorder.
CSG--crest stage gage.

Table (6): List of watersheds and flow measuring stations whose records are used in checking parameters (continued).

Station number[a]	Watershed	Area (sq mi)	Type of gage[b]	Length of record (years)
1772	Tusler Creek	90.2	CSG	10
1773	Redwater Cr. trib.	0.29	CSG	11
1773.5	S. Fk. Dry Ash Cr.	5.74	CSG	12
1774	McCune Creek	29.9	CSG	11
1830	Big Muddy Creek	850	CSG	9
1831	Box Elder Creek	9.40	CSG	11
1832	Box Elder Creek	19.9	CSG	10
1833	Spring Creek	7.05	CSG	12
1834	Spring Creek	16.9	CSG	12
2162	Wets Creek	8.14	CSG	12
2163	W. Buckeye Cr.	1.54	CSG	12
2165	Pryor Creek	435	CSG	42
2960	Rosebud Creek	1260	CSG	19
3082	Basin Creek trib.	0.14	CSG	12
3083	Basin Creek	10.9	CSG	12
3247	Sand Creek	10.6	CSG	12
3329	North Creek	0.68	CSG	13
3341	Wolf Creek	9.09	CSG	12
3364.5	Spring Creek	3.88	CSG	11

[a]U.S. Geological Survey designation.

[b]WSR--water stage recorder.
CSG--crest stage gage.

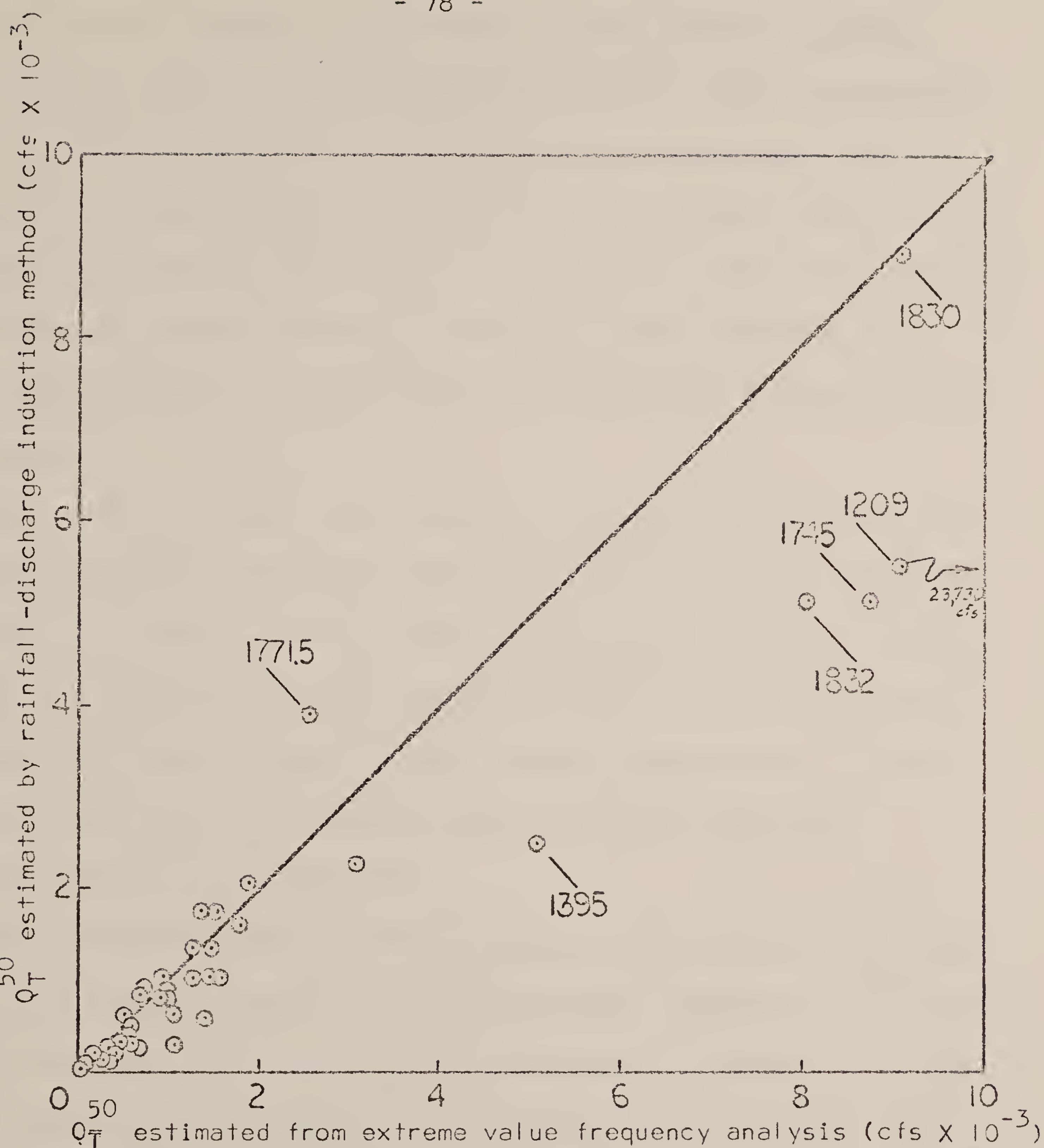


Figure (15): Difference in estimates of Q_T^{50} between the rainfall-discharge induction method and extreme value frequency analyses.

believed that the dominance of snow melt on the recurrence relation causes the true values of F for these watersheds to differ considerably from the average value used in Equation (27), and hence makes the estimate from Equation (27) invalid on these watersheds. The watersheds for which the estimates from the three analyses are similar are generally characterized as having a high proportion of rainfall-induced runoff events with the larger rainfall events often occurring during the snow-melt season.

Examination of Figure (15) and Figure (16) shows that, except for the numbered points (indicating watersheds discussed in the preceding paragraph) for which one or the other of the methods is believed to be invalid, the plotted points fall generally along the line of equality. This indicates that the three methods estimate the same set of values. In other words, the three methods estimate the same recurrence distribution for a given watershed.

The rainfall-discharge induction method predicts 50-year discharges larger than both the extreme value and log-normal estimates on 28 percent of the watersheds. The method predicts discharges intermediate in value between the other two estimates on 13 percent of the watersheds. The method predicts discharges smaller than either of the other estimates on 59 percent of the watersheds. (It was noted that the extreme value distribution predicts a higher discharge than does the log-normal distribution on all 46 watersheds.)

On some eastern Montana watersheds for which long streamflow records

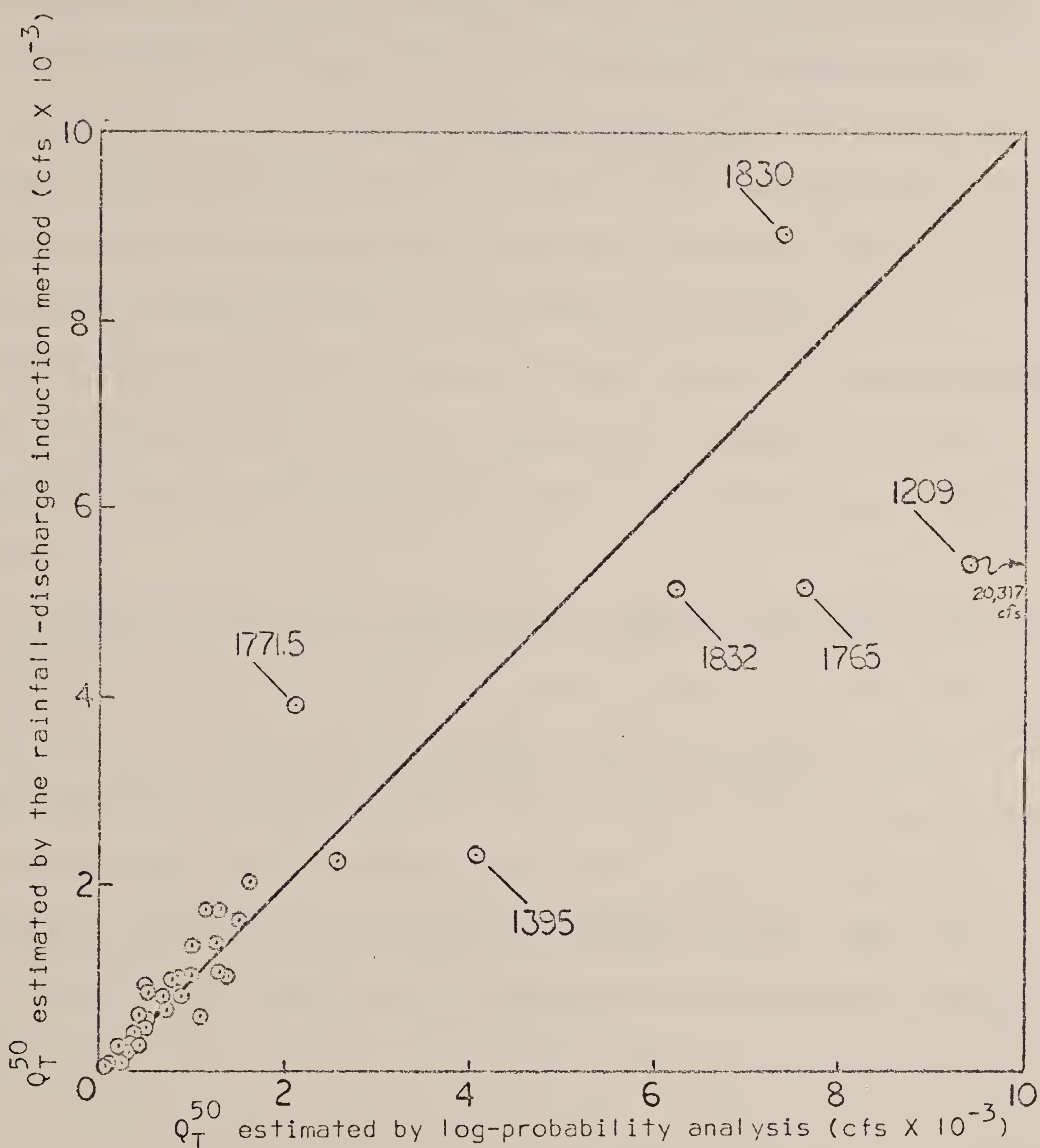


Figure (16): Difference in estimates of Q_T^{50} between the rainfall-discharge induction method and log-normal analyses.

are available, such as 1775, Redwater Creek, using only the most recent 15 years of record in extreme value and log-normal frequency analyses gives larger estimates of 50-year discharges than when the entire record is used. Short periods of record are sometimes non-representative with respect to the true distribution of discharges. The recent records on at least some watersheds, therefore, are perhaps non-representative. Non-representativeness of the samples on which the extreme value and log-normal estimates are made on the 46 watersheds is believed to be one reason that these estimates are often higher than the discharge-induction estimates.

The rainfall-discharge induction method estimates discharges apparently as well for watersheds as small as 0.2 square miles (e.g. Basin Creek Tributary) whose travel time is probably less than 12 hours as for larger watersheds, even though the values of the rainfall intensity ratio, R, are valid only for travel times larger than 12 hours. This fact implies that the error in using R for watersheds of travel times less than 12 hours is of lesser order of magnitude than the error in using an average value of F.

CHAPTER VI

APPLICATION AND EVALUATION OF THE RAINFALL-DISCHARGE INDUCTION METHOD

EXAMPLE OF METHOD APPLICATION

The rainfall-discharge induction method as developed above can be applied to a watershed with a short streamflow record, to estimate floods of various recurrence intervals as illustrated by the following example.

Watershed: Lone Man Coulee

Location: 5 miles south of Valier in Pondera County, Montana

Records: Annual floods for the years 1960 through 1966 listed by Boner (1967) were used here as listed in Table (7).

Determination:

$\bar{Q}_T$ = 339 cfs (Determined as the average of the annual floods listed in Table (7).)

R = 2.45 (Obtained from Figure (12).)

D = 1.60 (Obtained from Figure (13).)

F = 1.45 (Average value for Montana east of the continental divide, see p. 69.)

$$Q_T^{50} = \frac{DR}{F}\ \bar{Q}_T \quad \ldots\ldots\ldots\ldots \quad (27)$$

$$= \frac{(1.60)\ \ (2.45)}{(1.45)}\ (339) = 917 \text{ cfs}$$

Discharge values for recurrence intervals other than 50 years can be obtained by plotting $\bar{Q}_T$ (at recurrence interval 2.33 years) and Q_T^{50} (at recurrence interval 50 years) on extreme value probability paper

Table (7): Annual floods on Lone Man Coulee (after Boner, 1967).

Water Year	Discharge (cfs)
1960	32
1961	22
1962	15
1963	27
1964	1740
1965	400
1966	140
Mean:	339

and drawing a straight line through the two points. For the above example, for instance, Q_T^{25} = 780 cfs. For comparison, Q_T^{25} obtained by using the method of Boner and Omang (1967) is 350 cfs, while Q_T^{25} obtained by using the extreme value (Gumbel) method is 2220 cfs. (The Gumbel analysis is really meaningless, in this case, because of the short 7-year record, and the large, rare event of 1964.)

LIMITATIONS

The method was developed using watersheds from 31.4 to 684 square miles in area and tested on watersheds from 0.14 to 1805 square miles in area, with most of the watersheds being of less than 500 square miles in area. No difference in applicability was noted with watershed area for the watersheds tested.

Commonly, on large watersheds, snow melt events tend to be generally larger than rainfall events, since general rainstorms covering such large areas are rare. This characteristic has been shown to be true on Russian watersheds with climate similar to eastern Montana by Sokolov (1965). For this reason, it is recommended that this method be applied to watersheds of less than 500 square miles in area, until better limits of area-range can be established.

It was noted above that for some watersheds on which many large events are snow-melt caused the method gives results somewhat smaller than predicted from the extreme value analysis or the log-normal analysis. This is to be expected, since the method is based on rainfall-related parameters, and Equation (27) accounts for snow melt only to the extent

to which it interacts with rainfall. As a rule of thumb, if most of the annual floods of record, especially the larger ones, occur early in the season, inapplicability of the method is to be suspected. On the watersheds tested herein for which the method seems to be inapplicable, the largest 15 to 20 percent of the annual floods were caused by snow melt.

The chart given for estimating R is valid only for watersheds with travel times larger than 12 hours. Thus, the method should not be applied on watersheds with travel times shorter than 12 hours.

Values of R, D, and F are given for Montana east of the continental divide. These values should be used only within this region, since they may have quite different values in other areas.

RELIABILITY OF THE METHOD

The method presented in this thesis for predicting floods is considered to be as reliable as the extreme value recurrence fit or the log-normal analysis for discharge records 50 years in length and more reliable for shorter records.

It was shown above that the discharge induction method estimates the same set of values as the extreme value frequency method and the log-normal method. Although the three methods estimate the same set of values, the reliability of the three methods is different.

When the extreme value or log-normal method is used to predict floods of recurrence interval very much longer than the length of record used to fit the distribution, possible non-representativeness of the

sample makes the method unreliable. Linsley, Kohler and Paulhus (1958 caution against using the extreme value method or the log-normal method to predict floods of recurrence interval even approaching the record length. Applying this rule, a record longer than 50 years is required to predict the 50-year flood reliably. The rainfall-discharge induction method is much less sensitive to record length than the extreme value method or the log-normal method, since the streamflow is used only to predict the mean annual flood, while the 50-year flood is predicted from the factors R, D, and F which are based on much longer precipitation records (for the 58 watersheds studied herein, up to 63 years of rainfall records). On this basis, the rainfall-discharge induction method should be more reliable to predict the 50-year flood than either the extreme value method or the log-normal method when streamflow records are shorter than 50 years.

The rainfall-discharge induction method is sensitive, of course, to values used for R, D, and F. The isoplethal maps given for R and D are based on comparatively few values and may not reflect local conditions. Values of F computed for the individual watersheds vary as much as 30 percent from the mean value presented for use. This percentage, however, represents an extreme condition, and in most cases the error is less than this percentage implies. The scatter of values plotted in Figures (15) and (16) reflects this error as well as errors in the extreme value analysis and the log-normal analysis due to non-representativeness of samples and short record lengths.

SUGGESTIONS FOR FURTHER RESEARCH

It has been noted above that F varies quite widely, and has been represented by an average value, because of the lack of data and the complexity of the factor. It is believed that this factor can be improved by further research. Likewise, the factors R and D can be further refined as additional data becomes available.

This study approaches the problem of predicting peak discharges due to rainfall from rainfall parameters by separating the effects of snow melt from rainfall effects on hydrographs. It is believed that the same approach can be used to relate snow melt peaks to snow melt related parameters such as temperature, wind speed, etc. When this is completed, a factor to replace F in the discharge induction method may be formulated based on the discharge due to rain and the discharge due to snow melt. Research toward this goal is recommended.

CHAPTER VIII

SUMMARY

The rainfall-discharge induction method uses the mean annual discharge obtained from short streamflow records and three factors for which values are given herein to estimate floods of various recurrence intervals on Montana watersheds east of the continental divide. The method is applicable to watersheds of less than 500 square miles area for which large events most often occur during the snow melt season. The method is based on two assumptions which have been shown to be correct in Montana east of the continental divide:

1) The ratio of the 50-year rainfall intensities to the mean annual rainfall intensities for storms of durations equal to or greater than 12 hours is related to the frequency distribution of floods caused by rainfall on a watershed.

2) The frequency of floods caused by rainfall is related to the frequency of floods from all causes when the interaction between rainfall and snow melt is large.

The rainfall-discharge induction method is believed to be more reliable than extreme value frequency analysis or log-normal probability analysis for estimating floods from short streamflow records in Montana.

APPENDIX A

COMPUTER PROGRAMS

```
C     LOG NORMAL (LOG PROBABILITY) PROGRAM
      DIMENSION XF(38,7),CV(38),XK(7),YC(7)
    1 FORMAT (F6.3,7F5.2)
    2 FORMAT (I4,2F7.2)
      DO 21 J=1,38
   21 READ 1, CV(J),XF(J,1),XF(J,2),XF(J,3),XF(J,4),XF(J,5),XF(J,6),
     1XF(J,7)
   80 READ 2,NSTANO,YBAR,COV
      COFV=COV/100.
      I=0
   10 I=I+1
      IF(CV(I) .LT. COFV) GO TO 10
      IF(CV(I) .EQ. COFV .OR. I .LE. 1) GO TO 20
      DO 40 II=1,7
   40 XK(II)=XF(I-1, II)+(CV(I)-COFV)/(CV(I)-CV(I-1))*(XF(I,II)-
     1XF(I-1,II))
      GO TO 30
   20 DO 50 II=1,7
   50 XK(II)=XF(I,II)
   30 DO 60 N=1,7
   60 YC(N)=YBAR*(1.0 +XK(N)*COFV)
    3 FORMAT(7H STANO=,I4,5HYBAR=,F7.2,5HCOFV=,F7.3//)
      WRITE (108,3) NSTANO, YBAR,COFV
    4 FORMAT(57H TR=  1       2    2.33       5       20       50   100
     1 ,// 7F8.2/)
      WRITE (108,4) (YC(N),N=1,7)
      IF(NSTANO .NE. 0) GO TO 80
      CALL EXIT
      END
C     LOG PROBABILITY FACTORS. CARDS FOLLOW PROGRAM AS DATA.
C COEF.     RECURRENCE INTERVAL
C VAR.  1    2    2.33  5    20   50   100
 .000-2.33-0.00 0.14 0.84 1.64 2.03 2.33
 .033-2.25-0.02 0.12 0.84 1.67 2.09 2.40
 .067-2.18-0.04 0.10 0.83 1.70 2.14 2.47
 .100-2.11-0.06 0.09 0.82 1.72 2.19 2.55
 .136-2.04-0.07 0.08 0.81 1.75 2.25 2.62
 .166-1.98-0.09 0.06 0.80 1.77 2.30 2.70
 .197-1.91-0.10 0.05 0.79 1.79 2.35 2.77
 .230-1.85-0.11 0.04 0.78 1.81 2.40 2.84
 .262-1.79-0.13 0.02 0.77 1.82 2.44 2.90
 .292-1.74-0.14 0.01 0.76 1.84 2.48 2.97
 .324-1.68-0.15 0.00 0.75 1.85 2.52 3.03
 .351-1.63-0.16-0.01 0.73 1.86 2.56 3.09
 .381-1.58-0.17-0.02 0.72 1.87 2.60 3.15
 .409-1.54-0.18-0.03 0.71 1.88 2.64 3.21
```

.436	-1.49	-0.19	-0.04	0.69	1.88	2.67	3.26
.462	-1.45	-0.20	-0.05	0.68	1.89	2.70	3.31
.490	-1.41	-0.21	-0.06	0.67	1.89	2.73	3.36
.517	-1.38	-0.22	-0.08	0.65	1.89	2.75	3.40
.544	-1.34	-0.22	-0.08	0.64	1.89	2.77	3.44
.570	-1.31	-0.23	-0.09	0.63	1.89	2.79	3.48
.596	-1.28	-0.24	-0.10	0.61	1.89	2.81	3.52
.620	-1.25	-0.24	-0.10	0.60	1.89	2.83	3.55
.643	-1.22	-0.25	-0.11	0.59	1.89	2.85	3.59
.667	-1.20	-0.25	-0.11	0.58	1.88	2.87	3.62
.691	-1.17	-0.26	-0.12	0.57	1.88	2.89	3.65
.713	-1.15	-0.26	-0.12	0.56	1.88	2.90	3.67
.734	-1.12	-0.26	-0.13	0.55	1.87	2.91	3.70
.755	-1.10	-0.27	-0.13	0.54	1.87	2.92	3.72
.776	-1.08	-0.27	-0.14	0.53	1.86	2.92	3.74
.796	-1.06	-0.27	-0.14	0.52	1.86	2.93	3.76
.818	-1.04	-0.28	-0.15	0.51	1.85	2.94	3.78
.857	-1.01	-0.28	-0.15	0.49	1.84	2.96	3.81
.895	-0.98	-0.29	-0.16	0.47	1.83	2.97	3.84
.930	-0.95	-0.29	-0.17	0.46	1.81	2.98	3.87
.966	-0.92	-0.29	-0.17	0.44	1.80	2.99	3.89
1.000	-0.90	-0.29	-0.18	0.42	1.78	2.99	3.91
1.081	-0.84	-0.30	-0.19	0.39	1.75	2.99	3.93
1.155	-0.80	-0.30	-0.19	0.37	1.71	2.98	3.95

```
C     PEAK ANNUAL PRECIPITATION PROGRAM
C     VERSION 1 : FORMAT TO FIT ESSA DAILY PRECIPITATION DATA
      DIMENSION IMON(100), IYR(100), IDAY(100),DAYS(100)
    9 FORMAT (2X, I5, 4X, 2(12, 1H/), I2, 4X, F6.2)
    2 FORMAT (2X, I5, 4X, 2(I2, 1H/), I2, 4X, F6.2
     130H PEAK24 HOURS OF PRECIPITATION)
      XN = -1.
      P= 0.0
      DDAY = 0.0
      XDAYS = 0.0
C     DATA CONVERSION ROUTINE
    1 FORMAT (2X, I4, 3I2, 10X, 4A1,6X, 2II)
  200 FORMAT ( 27H  ACCUMULATED PRECIPITATION, 2(I2, 1H/), I2)
  210 FORMAT ( 14H  MISSING DATA, 2(I2, 1H/), I2)
  305 FORMAT ( I3)
  304 FORMAT ( 3I3, 1X, F5.0)
      U= 0.0
      READ 305,MM
      DO 900 M = 1, MM
      READ 304,  IMON(M), IDAY(M), IYR(M), DAYS(M)
  900 CONTINUE
    3 READ 1, NSTANO, NYR, NMON, NDAY,JJ4,JJ3,JJ2,JJ1, NRN, NSN
      J4 = JJ4/1000
      J3 = JJ3/1000
      J2 = JJ2/1000
      J1 = JJ1/1000
      IF (NSTANO) 77, 600, 77
   77 IF (J1 - 20) 110, 153, 120
  110 IF (J2) 130, 130, 140
  130 IF (J4) 150, 150, 122
C     BRANCH TO 120 WHEN PRECIP OCCURS
C     BRANCH TO 140 WHEN PRECIP ACCUMULATED
C     BRANCH TO 150 WHEN MISSING DATA
C     BRANCH TO 122 WHEN NO PRECIP OR TRACE
  120 N = (J1 - 70) + (J2 - 70)*10 + (J3 - 70)*100 + (J4 - 70)*1000
      YN = N
      XN = -1.0
      IF (U) 121, 121, 122
  121 DPREC = YN / 100.
      GO TO 190
  150 XN = 0.0
  122 U = 0.0
      GO TO 160
  153 XN=-1.0
      U=0.0
```

```
      GO TO 160
  140 XN = 2.0
      U = 1.0
  160 DPREC = 0.0
  190 CONTINUE
      IF (XN) 400, 155, 145
  155 PRINT 210, NMON, NDAY, NYR
      GO TO 400
  145 PRINT 200,NMON, NDAY, NYR
  400 CONTINUE
C     ROUTINE FOR LISTING PRECIP PERIODS
      DO 314 N = 1, MM
      IF (IYR(M) -NYR) 314, 312, 314
  312 IF (IMON(M) - NMON) 314, 313, 314
  313 IF (IDAY(M) - NDAY) 314, 315, 314
  315 XDAYS = DAYS(M)
  314 CONTINUE
  319 IF (XDAYS - DDAY) 325, 325, 320
  320 DDAY = DDAY + 1.0
      PRINT 9, NSTANO, NMON, NDAY, NYR, DPREC
      GO TO 330
  325 DDAY = 0.0
      XDAYS = 0.0
  330 CONTINUE
C     MAIN PROGRAM DETERMINES PEAK DAILY PRECIP
   27 IF (P) 5, 5, 25
    5 XPREC = DPREC
      LMON = NMON
      LDAY = NDAY
      LYR = NYR
      LSN = NSN
      LRN = NRN
      P = 1.0
      GO TO 3
   25 IF (DPREC - XPREC) 40, 40, 30
   30 XPREC = DPREC
      LMON = NMON
      LDAY = NDAY
      LYR = NYR
      LSN = NSN
      LRN = NRN
   40 CONTINUE
C     A CHECK FOR THE LAST DAY IN THE YEAR
      IF (NMON - 12) 90, 50, 50
   50 IF (NDAY - 31) 90, 70, 90
```

```
 70 PRINT 2, NSTANO, LMON, LDAY, LYR, XPREC
    PUNCH 2, NSTANO, LMON, LDAY, LYR, XPREC
    P = 0.0
    IF (LSN) 75,75, 80
  6 FORMAT ( 6H  SNOW)
  7 FORMAT ( 6H  RAIN)
 80 PRINT 6
 75 IF (LRN) 90, 90, 85
 85 PRINT 7
 90 GO TO 3
600 CALL EXIT
    END
```

```
C     PEAK ANNUAL PRECIPITATION PROGRAM
C     VERSION 2 : FORMAT TO FIT ESSA HOURLY PRECIPITATION DATA
C     DATA CONVERSION ROUTINE
      DIMENSION K (25), HRP(25), TOT (25), IMON(25), LDAY(25), LYR(25)
    1 FORMAT ( 2X, F4.0, 3I2, I1, 12A3)
  101 FORMAT (6X, 3I2, 1X, 12A3, 29X, I2)
  200 FORMAT ( 27H  ACCUMULATED PRECIPITATION, 2(I2, 1h/), I2)
  210 FORMAT ( 14H  MISSING DATA, 2(I2, 1H/), I2)
      IIU = 01
      DO 905 KK=1,24
  905 TOT(KK)=0.0
      U=0.0
    3 READ 1, STANO, NYR, NMON, NDAY,NCRD, (K(I), I=1,12)
      READ 101, KYR, KMON, KDAY,(K(I), I = 13,24), III
  810 FORMAT(19H CARDS OUT OF ORDER, 2(I2,1H/),I2 /
     134H LOAD ANOTHER YEAR AND CHECK CARDS)
      IF (KMON - NMON) 803,800,803
  800 IF (KDAY - NDAY ) 803,801,803
  801 IF(KYR - NYR) 803,802,802
  803 TYPE 810, NMON, NDAY, NYR
      PAUSE
      GO TO 902
  802 CONTINUE
      IF (STANO) 600, 600, 4
    4 DO 190  I=1,24
      K1=K(I) / 100
      KK=K1*100
      J1=K(I)-KK
      J3=K1/100
      KKK=J3*100
      J2=K1-KKK
C     BRANCH TO 120 WHEN PRECIP OCCURS
C     BRANCH TO 140 WHEN PRECIP ACCUMULATED
C     BRANCH TO 150 WHEN MISSING DATA
C     BRANCH TO 160 WHEN NO PRECIP OR TRACE
      IF(J1)110,110, 120
  110 IF(J2)130,130,140
  130 IF(J3)150,150,  123
  120 N=(J1-70)+(J2-70)*10+(J3-70)*100
      YN=N
      IF(U)121,121,122
  121 HRP(I)=YN/100.
      XN = -1.0
      GO TO 190
  122 U=0.0
  123 XN = -1.0
```

```
      GO TO 160
  150 XN = 0.0
      GO TO 160
  140 XN = 2.0
      U=1.0
  160 HRP(I)=0.0
  190 CONTINUE
      IF (XN) 400, 155, 145
  155 PRINT 210, NMON, NDAY, NYR
      GO TO 400
  145 PRINT 200,NMON, NDAY,NYR
  400 CONTINUE
C     MAIN PROGRAM - DETERMINES PEAK HOURLY PRECIP
      IF (STANO) 27,600, 27
   27 SUM = 0.0
      DO 21 KK = 1, 24
      N = 24 / KK
      GO TO (9,9,9,9,21,9,21,9,21,21,21,9,
     121, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 9), KK
    9 DO 20 LL = 1, N
      L = 1 + (LL - 1)*KK
      M = KK- 1 + L
      DO 5 J = L, M
    5 SUM = SUM + HRP(J)
      IF (SUM - TOT(KK))20, 20, 10
   10 TOT(KK)= SUM
      LMON(KK)= NMON
      LDAY(KK)= NDAY
      LYR(KK) = NYR
   20 SUM = 0.0
   21 CONTINUE
C     A CHECK FOR THE LAST DAY IN THE YEAR
   45 IF (NMON - 12) 55,50,50
   50 IF(III - 1) 55,70,55
    2 FORMAT ( 2X, I5  , 4X, 2(I2, 1H/), I2, 4X,  F6.2 ,
     15H PEAK, I2, 23H HOURS OF PRECIPITATION)
   70 NSTANO = STANO
      DO 902 NN = 1, 24
      GO TO (901,901,901,901,902,901,902,901,902,902,902,901,
     1902,902,902,902,902,902,902,902,902,902,902,901) ,NN
  901 PRINT 2, NSTANO,LMON(NN), LDAY(NN),LYR(NN),TOT(NN), NN
      PUNCH 2, NSTANO,LMON(NN), LDAY(NN),LYR(NN),TOT(NN), NN
  902 CONTINUE
      DO 903 KK=1,24
  903 TOT(KK)=0.0
```

```
 55 IF (IIU-NDAY) 60, 65, 60
  7 FORMAT ( 17H  CHECK DATA CARD, 2(I2,1H/), I2)
 60 PRINT 7 , MMON, NDAY, NYR
 65 IIU = III
 80 GO TO 3
600 CALL EXIT
    END
```

```
C     EXTREME VALUE (GUMBEL) PROBABILITY PROGRAM
      DIMENSION LYR(400),X(400),M(400),TP(400)
     1 , XNAME(4)  , TOT(400)
    1 FORMAT (5X, I5, 3F8.2)
    2 FORMAT (2X, I5, 4X, 6X      , I2,F10.2    , 5X, I2)
   13 FORMAT(79H STATION-OBS       MEAN-X      STD-DEV  PROB-ERR COEF-VAR
     1REC-TP1 REC-TP2 REC-TP3/)
   14 FORMAT ( 1H0, I5, I7, 2F12.2,F10.2,F9.1,3F8.2,/)
   15 FORMAT (3H 19, I3, F11.2,I6,F8.2)
   16 FORMAT (//34H GUMBEL POINTS FOR FREQUENCY GRAPH/)
   17 FORMAT(55H VALUE-B PROBABILITY  REC-INT    VALUES-X/  ERROR BOUND)
   18 FORMAT (F7.4,F12.4,F9.4,F12.3, 2F8.2)
   24 FORMAT (30H YEAR   VALUES-X ORDER REC-INT)
   30 FORMAT (1H0)
   31 FORMAT (34H PROB OBSR REC-TP1 REC-TP2 REC-TP3/)
  200 FORMAT ( 18H  RECORD TOO SHORT)
  900 READ 1        ,NUMB,TP1,TP2,TP3
      IF (NUMB) 23,23,901
  901 PRINT 31
      PRINT 1        ,NUMB,TP1,TP2,TP3
      PRINT 30
      DO 903 I = 1,NUMB
      READ 2, NSTANO, LYR(I) , TOT(I),  NN
  903 X(I) = TOT(I)
      FNUMB = NUMB
      K = NUMB - 1
      DO 7 I=1,K
      IP1 = I+1
      DO 7 J=IP1,NUMB
      IF (X(I)-X(J)) 6,7,7
    6 T = X(I)
      X(I) = X(J)
      X(J) = T
      T = LYR(I)
      LYR(I) = LYR(J)
      LYR(J) = T
    7 CONTINUE
      SUMX = 0.
      DO 8 I=1,NUMB
      M(I) = I
      FMI = M(I)
      TP(I) = (FNUMB + 1.)/FMI
    8 SUMX = SUMX + X(I)
      XBAR = SUMX/FNUMB
      SUSQ = 0.
```

```
      DO 9 I=1,NUMB
      SQDIF = (X(I)-XBAR)**2
    9 SUSQ = SUSQ + SQDIF
      SIG = SQRTF(SUSQ/(FNUMB - 1.))
      PE = .6745*SIG
      CV = 100.*SIG/XBAR
      IF ( NUMB -8) 50,51,52
   52 IF (NUMB- 9) 61,61,62
   62 IF (NUMB-12)71,71,72
   72 IF (NUMB-15)81,81,82
   82 IF (NUMB-20)91,91,92
   92 IF (NUMB-28)101,101,102
  102 IF (NUMB-42)111,111,112
  112 IF (NUMB-70)121,121,122
  122 IF (NUMB-150)131,131,132
   50 PRINT 200
   51 YN=0.48
      SN=0.90
      GO TO 300
   61 YN=0.49
      SN=0.93
      GO TO 300
   71 YN=0.50
      SN=0.965
      GO TO 300
   81 YN=0.51
      SN=1.01
      GO TO 300
   91 YN=0.52
      SN=1.045
      GO TO 300
  101 YN=0.53
      SN=1.08
      GO TO 300
  111 YN = 0.54
      SN=1.12
      GO TO 300
  121 YN=0.55
      SN=1.16
      GO TO 300
  131 YN=0.56
      SN=1.20
      GO TO 300
  132 YN=0.57
      SN=1.25
  300 CONTINUE
```

```
      P1 = 1./TP1
      P2 = 1./TP2
      P3 = 1./TP3
      B1 = -ALOG(-ALOG(1.-1./TP1))
      B2 = -ALOG(-ALOG(1.-1./TP2))
      B3 = -ALOG(-ALOG(1.-1./TP3))
      X1=B1*SIG/ SN +XBAR-  YN *SIG/ SN
      X2=B2*SIG/ SN +XBAR-  YN *SIG/ SN
      X3=B3*SIG/ SN +XBAR-  YN *SIG/ SN
      YDIF= SN /SIG*(X1  -XBAR)
      RSE=SQRTF(0.5236+1.1396*YDIF*3.14159/2.49949+1.10*
     1(YDIF*YDIF))
      XCU1  =X1  +RSE*SIG/( SN*FNUMB**.5)
      XCU2  =X2  +RSE*SIG/( SN*FNUMB**.5)
      XCU3  =X3  +RSE*SIG/( SN*FNUMB**.5)
      XCL3  =X3  -RSE*SIG/( SN*FNUMB**.5)
   40 XCL1  =X1  -RSE*SIG/( SN*FNUMB**.5)
      XCL2  =X2  -RSE*SIG/( SN*FNUMB**.5)
   10 PRINT 30
  301 FORMAT (5H PEAK, I2, 16H HOURS OF PRECIP    )
      PRINT 301, NN
      PRINT 13
      PRINT 14, NSTANO,NUMB,XBAR,SIG,PE,CV,TP1,TP2,TP3
      PRINT 30
      PRINT 24
      PRINT 15, (LYR(I),X(I),M(I),TP(I),              I=1,NUMB)
   38 PRINT 16
      PRINT 17
      PRINT 18, B1,P1,TP1,X1 ,XCU1,XCL1
      PRINT 18, B2,P2,TP2,X2,XCU2,XCL2
      PRINT 18, B3,P3,TP3,X3,XCU3,XCL3
      GO TO 900
   23 CALL EXIT
      END
```

APPENDIX B

PROCEDURES AND RESULTS

Table (1): Parameters of the extreme-value recurrence analysis for various weather stations.

Station number	Station name	Hours averaged	Mean annual rain during hours averaged (in.)	Coefficient of variation (percent)	50-year rain rain during hours averaged (in.)
466	Barber	24	0.99	28.5	1.92
780	Big Timber	24	1.36	48.0	3.26
807	Billings WB AP	24	1.33	49.5	3.47
877	Blackleaf	24	1.39	77.8	5.20
1088	Bredette	1	0.66	54.9	1.80
1088	Bredette	2	0.76	51.9	2.00
1088	Bredette	3	0.84	48.6	2.13
1088	Bredette	4	0.87	47.1	2.16
1088	Bredette	6	0.98	48.9	2.48
1088	Bredette	8	1.03	49.1	2.61
1088	Bredette	12	1.16	54.0	3.13
1088	Bredette	24	1.31	50.2	3.37
1127	Broadus	1	0.68	43.9	1.63
1127	Broadus	2	0.76	49.2	1.94
1127	Broadus	3	0.84	52.8	2.25
1127	Broadus	4	0.88	50.7	2.28
1127	Broadus	6	0.97	48.5	2.46
1127	Broadus	8	1.01	54.0	2.72
1127	Broadus	12	1.05	45.3	2.56
1127	Broadus	24	1.20	50.3	3.11
1169	Brockway	24	1.86	54.6	5.73
1342	Bynum 4SSE	24	1.27	72.9	4.55
1737	Choteau	1	0.58	46.3	1.42
1737	Choteau	2	0.66	39.2	1.47
1737	Choteau	3	0.68	38.1	1.50
1737	Choteau	4	0.76	37.0	1.66
1737	Choteau	6	0.80	38.7	1.78
1737	Choteau	8	0.93	44.5	2.22
1737	Choteau	12	1.09	54.7	2.96
1737	Choteau	24	1.31	54.9	3.56
1765	Circle 7N	24	1.46	51.5	3.92
1974	Conrad	24	1.36	181.6	3.56
2477	Dovetail 1N	1	0.49	63.5	1.47
2477	Dovetail 1N	2	0.59	57.9	1.67
2477	Dovetail 1N	3	0.68	66.4	2.11

Table (1): Parameters of the extreme value recurrence analysis for various weather stations (continued).

Station number	Station name	Hours averaged	Mean annual rain during hours averaged (in.)	Coefficient of variation (percent)	50-year rain rain during hours averaged (in.)
2477	Dovetail 1N	4	0.72	59.5	2.08
2477	Dovetail 1N	6	0.85	78.6	2.97
2477	Dovetail 1N	8	0.95	69.6	3.02
2477	Dovetail 1N	12	1.04	65.7	3.19
2477	Dovetail 1N	24	1.22	59.4	3.51
2571	Dupuyer	24	1.16	37.6	2.66
2689	Ekalaka	24	1.64	53.9	4.52
2820	Ethridge	24	1.49	39.3	3.58
2996	Fishtail	24	1.53	45.8	4.02
3113	Fort Benton	24	1.43	49.2	3.72
3557	Glasgow WB AP	24	1.16	59.9	3.43
3751	Great Falls WB AP	24	1.45	37.0	3.20
3939	Harlowton	24	1.08	62.5	3.35
3996	Havre WB AP	---[a]	---[a]	---[a]	---[a]
4055	Helena WB AP	---[a]	---[a]	---[a]	---[a]
4358	Hysham	24	1.27	45.8	3.18
4538	Judith Gap	24	1.17	89.5	4.71
4558	Kalispell WB AP	---[a]	---[a]	---[a]	---[a]
4663	Kings Hill	1	0.39	35.4	0.83
4663	Kings Hill	2	0.48	48.0	1.07
4663	Kings Hill	3	0.57	32.1	1.14
4663	Kings Hill	4	0.60	31.5	1.20
4663	Kings Hill	6	0.72	31.0	1.44
4663	Kings Hill	8	0.82	35.3	1.72
4663	Kings Hill	12	0.92	33.4	1.90
4663	Kings Hill	24	1.17	33.1	2.38
4985	Lewistown AP	24	1.64	37.6	3.64
5086	Livingston FAA AP	1	0.40	63.1	1.19
5086	Livingston FAA AP	2	0.47	48.9	1.19
5086	Livingston FAA AP	3	0.53	43.9	1.27

[a]Values of R were computed directly from values given by U.S. Weather Bureau (1956).

Table (1): Parameters of the extreme value recurrence analysis for various weather stations (continued).

Station number	Station name	Hours averaged	Mean annual rain during hours averaged (in.)	Coefficient of variation (percent)	50-year rain rain during hours averaged (in.)
5086	Livingston FAA AP	4	0.54	42.1	1.26
5086	Livingston FAA AP	6	0.63	33.9	1.30
5086	Livingston FAA AP	8	0.66	33.6	1.36
5086	Livingston FAA AP	12	0.75	30.2	1.46
5086	Livingston FAA AP	24	0.88	34.1	1.83
5235	Loweth	24	0.83	21.8	1.48
5337	Malta	24	1.18	26.4	2.20
5387	Martinsdale	1	0.48	64.9	1.46
5387	Martinsdale	2	0.53	61.3	1.55
5387	Martinsdale	3	0.57	55.4	1.58
5387	Martinsdale	6	0.70	51.4	1.84
5387	Martinsdale	8	0.73	41.0	1.66
5387	Martinsdale	12	0.80	37.3	1.75
5387	Martinsdale	24	0.93	34.6	1.95
5603	Melville 4W	24	1.36	26.7	2.75
5685	Miles City	---[a]	---[a]	---[a]	---[a]
5745	Missoula	---[a]	---[a]	---[a]	---[a]
6426	Pendroy	24	1.66	51.9	4.58
6918	Red Lodge	24	1.74	36.4	3.82
7214	Roundup	24	1.13	41.2	2.65
7560	Sidney	24	1.27	33.1	1.98
8169	Terry 25NNW	1	0.76	53.5	2.04
8169	Terry 25NNW	2	0.88	54.7	2.39
8169	Terry 25NNW	3	0.94	52.4	2.49
8169	Terry 25NNW	4	0.98	53.1	2.61
8169	Terry 25NNW	6	1.07	50.3	2.78
8169	Terry 25NNW	8	1.14	47.5	2.84
8169	Terry 25NNW	12	1.20	44.0	2.85
8169	Terry 25NNW	24	1.38	44.4	3.30
8501	Valier	24	1.46	49.9	3.90

[a] Values of R were computed directly from values given by U.S. Weather Bureau (1956).

Table (1): Parameters of the extreme value recurrence analysis for various weather stations (continued).

Station number	Station name	Hours averaged	Mean annual rain during hours averaged (in.)	Coefficient of variation (percent)	50-year rain rain during hours averaged (in.)
8597	Virginia City	24	1.01	27.2	1.92
8927	White Sulphur Springs	24	1.00	37.8	2.10
8957	Wilbaux 2E	24	1.59	47.3	4.05
9018	Wilsall	24	0.94	34.3	2.08
Average:		---	---	43.8	---

Table (2): Parameters of the recurrence analysis for watersheds used in the runoff analysis.

		Peak discharge due to rainfall			Peak discharge due to snow melt			Total peak discharge			
Station number[a]	Watershed	Mean annual (cfs)	Coef. of var. (%)	50-year (cfs)	Mean annual (cfs)	Coef. of var. (%)	50-year (cfs)	Mean annual (cfs)	Coef. of var. (%)	50-year (cfs)	F
Watersheds used in first phase of the runoff analysis:											
770	Sheep Cr. (near White Sulphur Springs)	50.5	71.0	177[b]	138	42.3	344[b]	201	41.6	498[b]	1.41
1155	N. Fk. Musselshell R.	21.1	75.3	70.9[b]	89.4	109.1	394[b]	111	83.7	402[b]	0.93
1185	S. Fk. Musselshell R.	344	78.2	1288[b,c]	620[c]	39.5	1381[b,c]	868[c]	37.3	1884[b,c]	1.72
2005	Sweetgrass Cr.	446	104.9	1852[b]	712	34.0	1441[b]	1069	41.6	2406[b]	1.84
Watersheds used in second phase of the runoff analysis:											
375	Madison River	304	44.4	742[b]	---	---	---	1329	22.9	2242[b]	1.45
1280	Box Elder Cr.	1827	133.8	10,037[b]	---	---	---	2077	118.1	10,314[b]	1.85
1545	Peoples Creek	279	95.2	1138[b]	---	---	---	1121	96.9	4637[b]	C.99
1775	Redwater Creek	1103	154.2	6214[b]	---	---	---	1898	98.8	7530[b]	1.42

[a]U.S. Geological Survey designation.
[b]Extreme value (Gumbel) estimate.
[c]Adjusted for irrigation above measuring station--150cfs on mean annual flood, 275 cfs on 50-year flood.

Table (2): Parameters of the recurrence analysis for watersheds used in the runoff analysis (continued).

		Peak discharge due to rainfall			Peak discharge due to snow melt			Total peak discharge			
Station number[a]	Watershed	Mean annual (cfs)	Coef. of var. (%)	50-year (cfs)	Mean annual (cfs)	Coef. of var. (%)	50-year (cfs)	Mean annual (cfs)	Coef. of var. (%)	50-year (cfs)	F
Watersheds used in second phase of the runoff analysis (continued):											
1780	M. Fk. Poplar R.	208	171.4	1280	---	---	---	1277	177.3	8076[b]	0.97
2890	Little Bighorn R.	249	81.6	905	---	---	---	1067	47.3	2644[b]	1.47
3365	Beaver Creek	429	171.1	2803	---	---	---	1208	76.7	4101[b]	1.65

[a]U.S. Geological Survey designation.
[b]Extreme value (Gumbel) estimate.

Table (3): Parameters of the recurrence analysis for watersheds used to test the rainfall-discharge induction method.

Station number[a]	Watershed	Total peak discharge: Mean annual (cfs)	Coef. of var. (percent)	50-year[b] (cfs)	50-year[c] (cfs)
135	Sheep Cr. (near Dell)	354	53.8	994	878
155	Grasshopper Cr.	446	72.7	1420	1387
465	E. Gallatin R.	320	43.7	880	697
470	Bear Canyon Cr.	164	66.8	552	481
760	Newland Creek	17.8	84.8	68.5	62.2
893	Sun River trib.	124	119.3	435[d]	382
1003	Lone Man Coulee	339	186.4	682[d]	501
1022	Marias R. trib.	40.9	217.4	354	268
1023	Marias R. trib. No. 2	10.3	125.6	55.8	42.7
1206	Antelope Cr. trib.	3.0	134.2	17.5	13.2
1207	Antelope Cr. trib.	77.3	94.9	336	296
1208	Bacon Creek	225	145.9	1383	1094
1209	Antelope Cr.	2090	308.1	23,730	20,377
1289	Box Elder Cr. trib.	155	71.7	546	477
1295	McDonald Cr.	369	86.9	1372	1321
1297	Gorman Coulee	95.2	124.0	512	392
1298	Gorman Coulee trib.	52.4	107.0	250	220

[a]U.S. Geological Survey designation.
[b]Extreme value (Gumbel) estimate.
[c]Log-normal (log probability) estimate.
[d]Adjusted for the event of June 1964, a very rare event.

Table (3): Parameters of the recurrence analysis for watersheds used to test the rainfall-discharge induction method (continued).

Station number[a]	Watershed	Total peak discharge: Mean annual (cfs)	Coef. of var. (percent)	50-year[b] (cfs)	50-year[c] (cfs)
1306	Cat Creek	190	123.5	1053	781
1308	Second Cr. trib.	71.5	182.9	533	413
1395	Big Sandy Cr.	827	159.2	4939	4093
1552	Alkali Cr.	346	92.8	1525	1300
1553	Disjardin Coulee	97.0	113.9	487	427
1554	S. Fk. Taylor Cr.	29.1	114.6	147	129
1765	Wolf Creek	1032	232.9	8697	7632
1770.5	E. Fk. Duck Cr.	219	85.1	878	769
1771	Duck Creek	441	88.6	1820	1599
1771.5	Redwater Creek	843	55.2	2486	2138
1772	Tusler Creek	186	67.6	630	549
1773	Redwater Cr. trib.	41.8	166.6	287	225
1773.5	S. Fk. Dry Ash Cr.	40.4	733	145	126
1774	McCune Creek	226	125.7	1230	986
1830	Big Muddy Cr.	1731	1308	9062	7404
1831	Box Elder Cr.	119	88.4	492	432
1832	Box Elder Cr.	1163	166.6	8000	6256
1833	Spring Creek	49.1	132.9	279	212
1834	Spring Creek	262	105.7	1240	1090

[a] U.S. Geological Survey designation.
[b] Extreme value (Gumbel) estimate.
[c] Log-normal (log probability) estimate.

Table (3): Parameters of the recurrence analysis for watersheds used to test the rainfall-discharge induction method (continued).

		Total peak discharge			
number[a]	Watershed	Mean annual (cfs)	Coef. of var. (percent)	50-year[b] (cfs)	50-year[c] (cfs)
2162	Wets Creek	118	132.0	667	533
2163	W. Buckeye Cr.	93.6	83.3	369	324
2165	Pryor Creek	750	82.0	2597	2570
2960	Rosebud Cr.	301	64.8	932	859
3082	Basin Cr. trib.	52.2	209.8	439	351
3083	Basin Creek	412	92.6	1756	1545
3247	Sand Creek	56.2	134.8	324	258
3329	North Creek	312	104.0	1403	1283
3341	Wolf Creek	396	75.9	1403	1283
3364.5	Spring Creek	111	102.8	515	454

[a]U.S. Geological Survey designation.
[b]Extreme value (Gumbel) estimate.
[c]Log-normal (log probability) estimate.

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